

# Introduction to ssGBLUP

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Our genetic model is:

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{W}\mathbf{u} + \mathbf{e}$$

$\mathbf{y}$  = vector of phenotypes

$\mathbf{X}\mathbf{b}$  = matrix relating  $\mathbf{y}$  with fixed effects in  $\mathbf{b}$

$\mathbf{W}\mathbf{u}$  = matrix relating  $\mathbf{y}$  with random effects in  $\mathbf{u}$ ,  $Var(\mathbf{u}) = \mathbf{A}\sigma_u^2$

$\mathbf{e}$  = vector of random errors,  $Var(\mathbf{e}) = \mathbf{I}\sigma_e^2$

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{A}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix}$$

Best: minimizes MSE

Linear: linear function of the data

Unbiased:  $E(u) = E(\hat{u})$

Prediction: for random effects

$$u_i = u_{s\_i} + u_{d\_i}$$

$$p(\mathbf{y}, \mathbf{u}) = p(\mathbf{u}|\mathbf{y})p(\mathbf{y}) = p(\mathbf{y}|\mathbf{u})p(\mathbf{u})$$



## BLUP-based methods

### That BLUP Is a Good Thing: The Estimation of Random Effects

G. K. Robinson

- Unbalanced data and information from relatives

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{A}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix}$$

$h^2$  is high

$\lambda$  goes to zero

$\mathbf{A}^{-1}\lambda$  goes to zero

“Relationships don’t matter”



$$\lambda = \frac{1 - h^2}{h^2}$$

$h^2$  is low

$\lambda$  goes to infinity

$\mathbf{A}^{-1}\lambda$  goes to infinity

“Relationships matter a lot”



# BLUP-based methods

Theor Appl Genet (1983) 67:25–33



## Genetic polymorphism in varietal identification and genetic improvement \*

M. Soller<sup>1</sup> and J. S. Beckmann<sup>2</sup>

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<sup>2</sup> Institute of Field and Garden Crops, Agricultural Research Organization, The Volcani Center 50250 Bet Dagan, Israel

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**Summary.** New sources of genetic polymorphisms promise significant additions to the number of useful genetic markers in agricultural plants and animals, and prompt this review of potential applications of polymorphic genetic markers in plant and animal breeding. Two major areas of application can be distinguished. The first is based on the utilization of genetic markers to determine genetic relationships. These applications include varietal identification, protection of breeder's rights, and parentage determination. The second area of application is based on the use of genetic markers to identify and map loci affecting quantitative traits, and to monitor these loci during introgression or selection programs. A variety of breeding applications based on

- Use of DNA polymorphisms as genetic markers
- Construct genetic relationships
- Parentage determination
- Identification of QTL
- RFLP (expensive)



Soller and Beckman, 1982

# BLUP-based methods

## CROP BREEDING, GENETICS & CYTOLOGY

### Prediction of Maize Single-Cross Performance Using RFLPs and Information from Related Hybrids

Rex Bernardo\*

#### ABSTRACT

Methods for predicting hybrid yield would facilitate the identification of superior maize (*Zea mays* L.) single crosses. Best linear unbiased prediction of the performance of single crosses, based on (i) restriction fragment length polymorphism (RFLP) data on the parental inbreds and (ii) yield data on a related set of single crosses, was evaluated. Yields of  $m$  single crosses were predicted as  $y_M = C V^{-1} y_P$ , where:  $y_M = m \times 1$  vector of predicted yields of missing (i.e., no yield data available) single crosses;  $C = m \times n$  matrix of genetic covariances between the missing and predictor hybrids;  $V = n \times n$  matrix of phenotypic variances and covariances among predictor hybrids; and  $y_P = n \times 1$  vector of predictor hybrid yields corrected for trial effects. From a set of 54 single crosses, made between six Iowa Stiff Stalk Synthetic (SSS) and nine non-SSS inbreds, 100 different sets of  $n = 10, 15, 20, 25$ , or 30 predictor hybrids were chosen at random. Pooled correlations between predicted and observed yields of the remaining  $(54 - n)$  hybrids ranged from 0.654 to 0.800. The correlations were slightly higher when dominance variance was included in the model or when coefficients of coancestry were determined from RFLP rather than pedigree data. The correlations remained relatively stable across different, arbitrary values of genetic variances. The results suggested that single-cross yield can be predicted effectively based on parental RFLP data and yields of a related set of hybrids.

marker dissimilarity between parents. Restriction fragment length polymorphisms have been found useful for assigning inbreds to heterotic groups as well as for determining relationships among inbreds in the same heterotic group (Smith et al., 1990; Melchinger et al., 1991; Dudley et al., 1991; Hogan and Dudley, 1992; Bernardo, 1993). But in theoretical (Bernardo, 1992; Charcosset et al., 1991) as well as empirical studies using RFLPs (Godshalk et al., 1990; Melchinger et al., 1990; Dudley et al., 1991), the correlations between single-cross yield and molecular marker dissimilarity between parents have been too low to be of any predictive value.

Although yield data may not be available for all possible single-cross combinations among available inbreds, some of these combinations already may have been evaluated by the breeder. For example, yield data may be available for 200 out of 2500 possible hybrids between 50 inbreds from X and 50 inbreds from Y. If information on the RFLP or pedigree relationships among the 100 parental inbreds is available, by best linear unbiased prediction (BLUP) (Henderson, 1975; 1985) the yield data on the 200 tested hybrids may be used to predict the yields of the remaining 2300 untested hybrids. The BLUP procedure, usually assuming an additive and intrapop-

- Use of DNA polymorphisms as genetic markers
- Construct genetic relationships
- Parentage determination
- Identification of QTL
- RFLP (expensive)



Bernardo, 1994

# BLUP-based methods

## articles

### Initial sequencing and analysis of the human genome

International Human Genome Sequencing Consortium\*

\* A partial list of authors appears on the opposite page. Affiliations are listed at the end of the paper.

The human genome holds an extraordinary trove of information about human development, physiology, medicine and evolution. Here we report the results of an international collaboration to produce and make freely available a draft sequence of the human genome. We also present an initial analysis of the data, describing some of the insights that can be gleaned from the sequence.

The rediscovery of Mendel's laws of heredity in the opening weeks of the 20th century<sup>1-3</sup> sparked a scientific quest to understand the nature and content of genetic information that has propelled biology for the last hundred years. The scientific progress made falls naturally into four main phases, corresponding roughly to the four quarters of the century. The first established the cellular basis of heredity: the chromosomes. The second defined the molecular basis of heredity: the DNA double helix. The third unlocked the informational basis of heredity, with the discovery of the biological mechanism by which cells read the information contained in genes and with the invention of the recombinant DNA technologies of cloning and sequencing by which scientists can do the same.

The last quarter of a century has been marked by a relentless drive to decipher first genes and then entire genomes, spawning the field of genomics. The fruits of this work already include the sequences of 599 viruses and viroids, 205 naturally occurring plasmids, 185 organelles, 31 eubacteria, seven archaea, two animals and one plant.

Here we report the results of a collaboration involving 20 from the United States, the United Kingdom, Japan, Germany and China to produce a draft sequence of the genome. The draft genome sequence was generated from a map covering more than 96% of the euchromatic part of the genome and, together with additional sequence in public data, it covers about 94% of the human genome. The sequence produced over a relatively short period, with coverage rising about 10% to more than 90% over roughly fifteen months. Sequence data have been made available without restriction, updated daily throughout the project. The task ahead is to produce a finished sequence, by closing all gaps and resolving all ambiguities. Already about one billion bases are in final form and the bringing the vast majority of the sequence to this standard straightforward and should proceed rapidly.

The sequence of the human genome is of interest in many respects. It is the largest genome to be extensively sequenced, being 25 times as large as any previously sequenced genome, eight times as large as the sum of all such genomes. It is the first vertebrate genome to be extensively sequenced. And, uniquely, it is the genome of our own species.

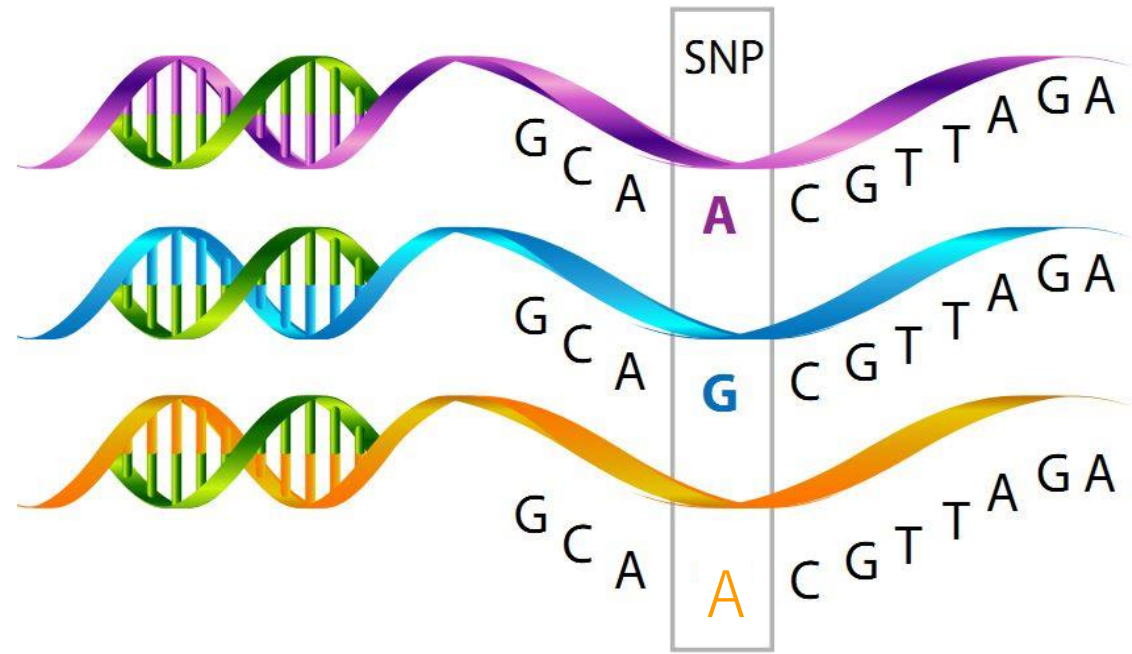
Much work remains to be done to produce a complete sequence, but the vast trove of information that has been made available through this collaborative effort allows a global perspective on the human genome. Although the details will change as the sequence is finished, many points are already clear.

coordinate regulation of the genes in the clusters.

● There appear to be about 30,000–40,000 protein-coding genes in the human genome—only about twice as many as in worm or fly. However, the genes are more complex, with more alternative splicing generating a larger number of protein products.

● The full set of proteins (the 'proteome') encoded by the human genome is more complex than those of invertebrates. This is due in part to the presence of vertebrate-specific protein domains and motifs (an estimated 7% of the total), but more to the fact that vertebrates appear to have arranged pre-existing components into a richer collection of domain architectures.

● Hundreds of human genes appear likely to have resulted from



<http://neuroendoimmune.files.wordpress.com/2014/03/snp.png>

Mutation < 1% < SNP

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## Prediction of Total Genetic Value Using Genome-Wide Dense Marker Maps

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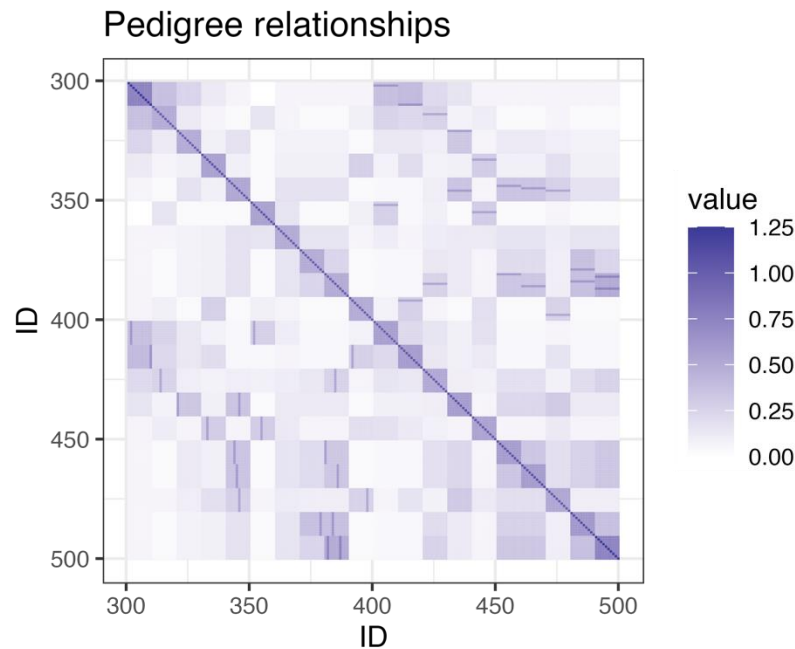


## BLUP-based methods

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{G}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix}$$

Only for genotyped animals

- Better Mendelian sampling tracking

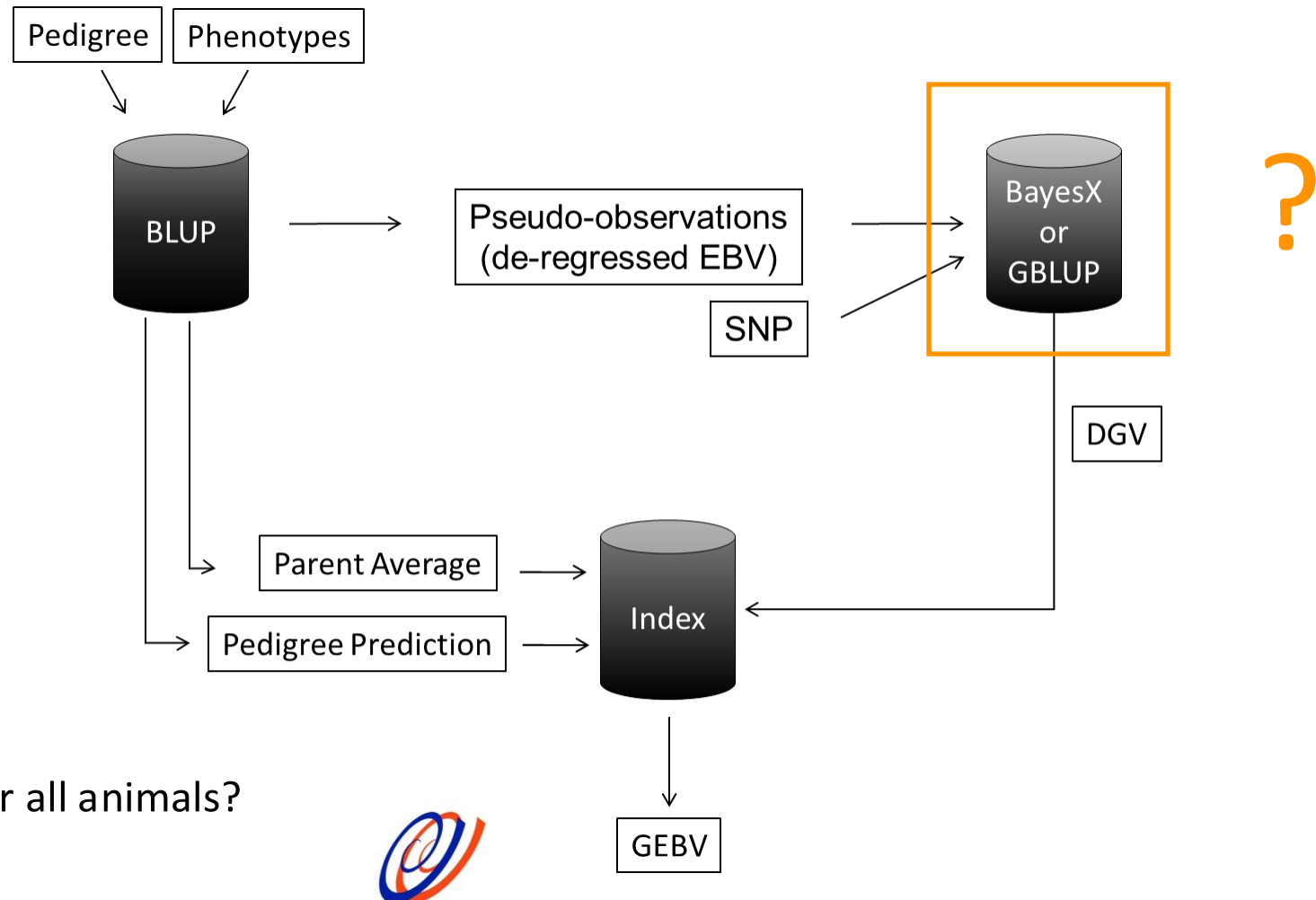


Bernardo, 1994  
Nejati-Javaremi et al. (1997)  
VanRaden, 2008

Lourenco et al. (2015)

## BLUP-based methods

- In practice, not all individuals are genotyped

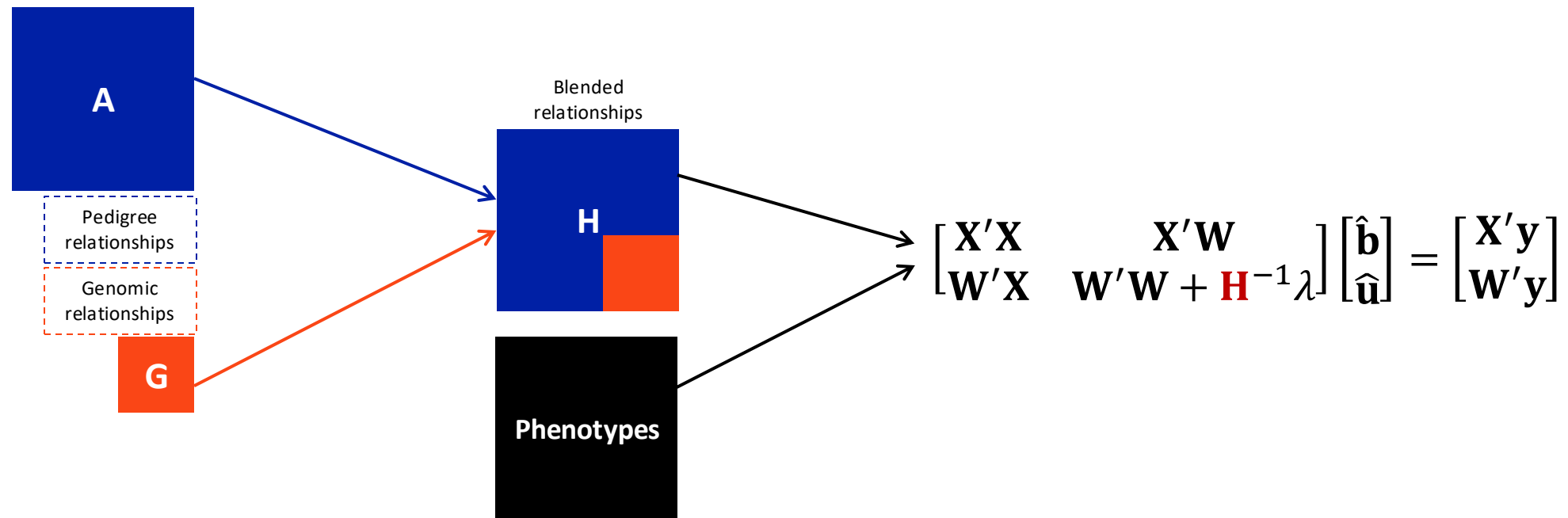


- How to obtain covariances for all animals?



## BLUP-based methods

- In practice, not all individuals are genotyped



- How to obtain covariances for all animals?



## BLUP-based methods

- Genomic evaluation would be simpler if all individuals were genotyped
- What should be done when there are genotyped and non-genotyped individuals?
  - SNPs are capturing relationships
  - Pedigrees give information about relationships
  - Genomic and pedigree relationships can be combined in

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix}$$

Non-genotyped

Genotyped

$$\mathbf{H} = \mathbf{A} + \begin{bmatrix} 0 & 0 \\ 0 & -\mathbf{A}_{22} \end{bmatrix}$$



## BLUP-based methods

- $\mathbf{A}$  is the expectation of realized or observed relationships
- Consider  $\mathbf{A}$  as *prior* and  $\mathbf{G}$  as *observed* relationships, then construct *posterior* relationships

$$p(\mathbf{u}_2) = N(\mathbf{0}, \mathbf{G}\sigma_u^2)$$

$$p(\mathbf{u}_1|\mathbf{u}_2) = N(\mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{u}_2, \mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21})$$

$$p(\mathbf{u}_1, \mathbf{u}_2) = p(\mathbf{u}_1|\mathbf{u}_2)p(\mathbf{u}_2)$$

$$\text{Var} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix} = \mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21} + \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{G}\mathbf{A}_{22}^{-1}\mathbf{A}_{21} & \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{G} \\ \mathbf{G}\mathbf{A}_{22}^{-1}\mathbf{A}_{21} & \mathbf{G} \end{bmatrix}$$



## BLUP-based methods

$$\text{Var} \begin{bmatrix} \mathbf{u}_1 \\ \mathbf{u}_2 \end{bmatrix} = \mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \mathbf{H}_{12} \\ \mathbf{H}_{21} & \mathbf{H}_{22} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_{11} - \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{A}_{21} + \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{G}\mathbf{A}_{22}^{-1}\mathbf{A}_{21} & \mathbf{A}_{12}\mathbf{A}_{22}^{-1}\mathbf{G} \\ \mathbf{G}\mathbf{A}_{22}^{-1}\mathbf{A}_{21} & \mathbf{G} \end{bmatrix}$$

But ... we need  $\mathbf{H}^{-1}$



## BLUP-based methods

Surprisingly...

$$\mathbf{H}^{-1} = \mathbf{A}^{-1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}^{-1} - \mathbf{A}_{22}^{-1} \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{H}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix}$$



## BLUP-based methods

Animal	Sire	Dam
1	0	0
2	0	0
3	1	2
4	1	2

**A**

$$\begin{bmatrix} 1.0 & 0.0 & 0.5 & 0.5 \\ . & 1.0 & 0.5 & 0.5 \\ . & . & 1.0 & 0.5 \\ . & . & . & 1.0 \end{bmatrix}$$

**G**

$$\begin{bmatrix} 1.0 & 0.52 \\ . & 1.0 \end{bmatrix}$$

**H**

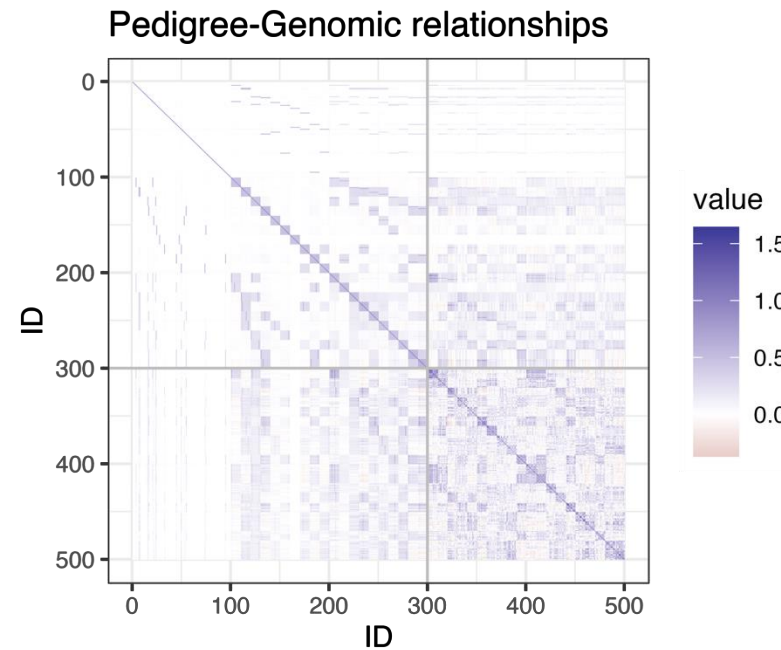
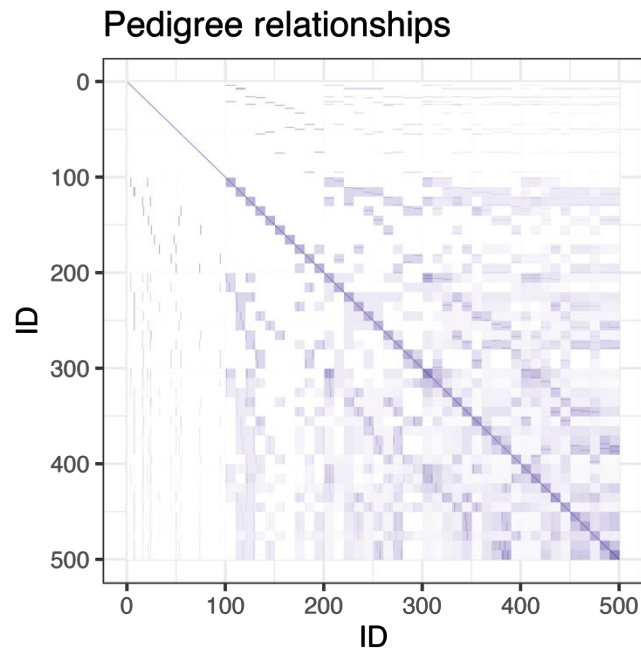
$$\begin{bmatrix} 1.004 & 0.0 & 0.507 & 0.507 \\ . & 1.004 & 0.507 & 0.507 \\ . & . & 1.0 & 0.52 \\ . & . & . & 1.0 \end{bmatrix}$$



## BLUP-based methods

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{H}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix}$$

- Projection of genomic relationships on the rest of individuals
- Bayesian update of  $\mathbf{A}$  based on new information from  $\mathbf{G}$





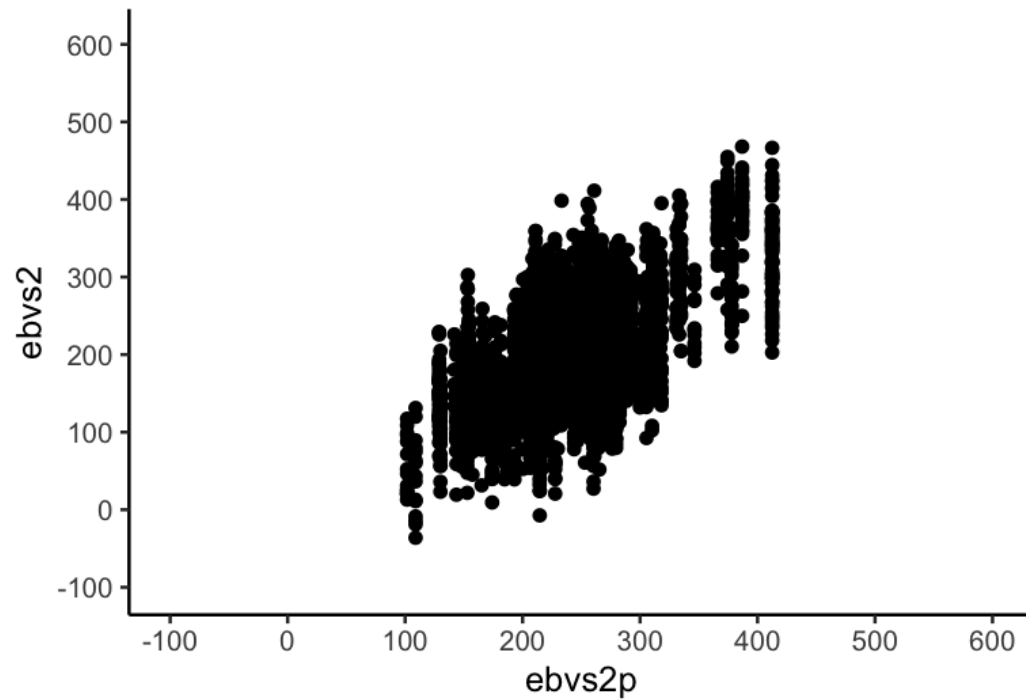
## BLUP-based methods

- $$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{A}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix}$$
  - **A**
    - Contains expected relationships
    - It is limited by the pedigree depth and completeness
    - Depends on the accuracy of recording pedigrees
- $$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{G}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix}$$
  - **G**
    - Contains the number of shared alleles between animals weighted by heterozygosity
    - There are no limitations regarding the number of past generations
    - It depends on allele frequency and quality of genomic data
- $$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{H}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix}$$
  - **H**
    - Projection of genomic relationships on the ungenotyped individuals
    - Bayesian updating of **A** based on new information from **G**

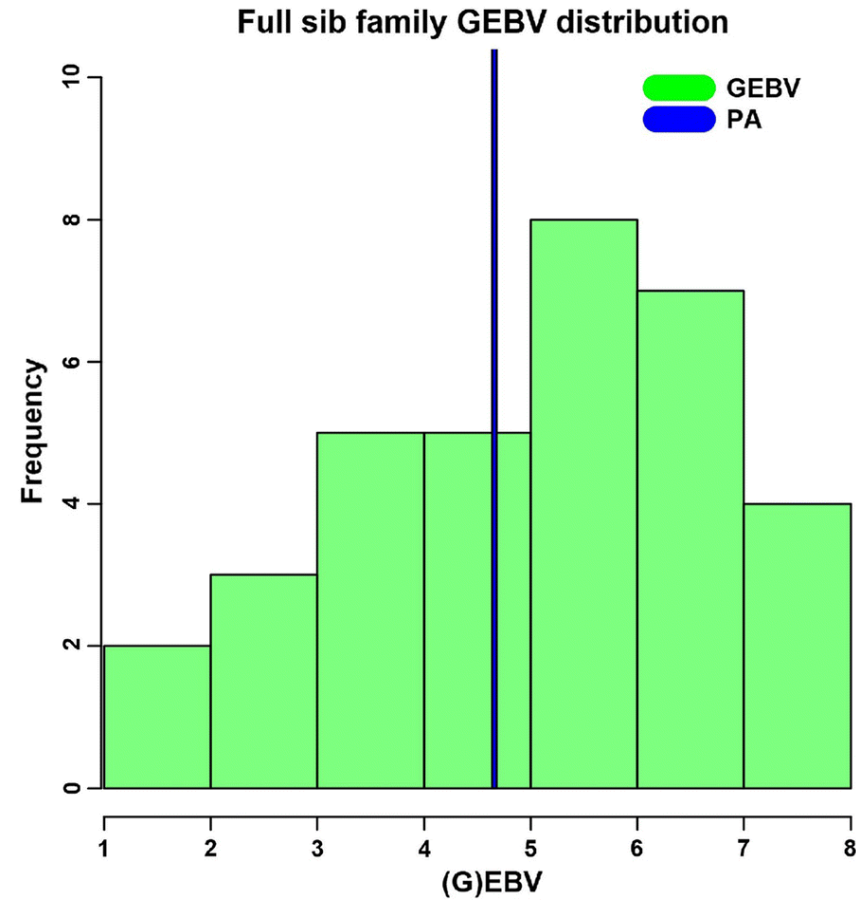


## BLUP-based methods

- Pedigree BLUP



## BLUP-based methods

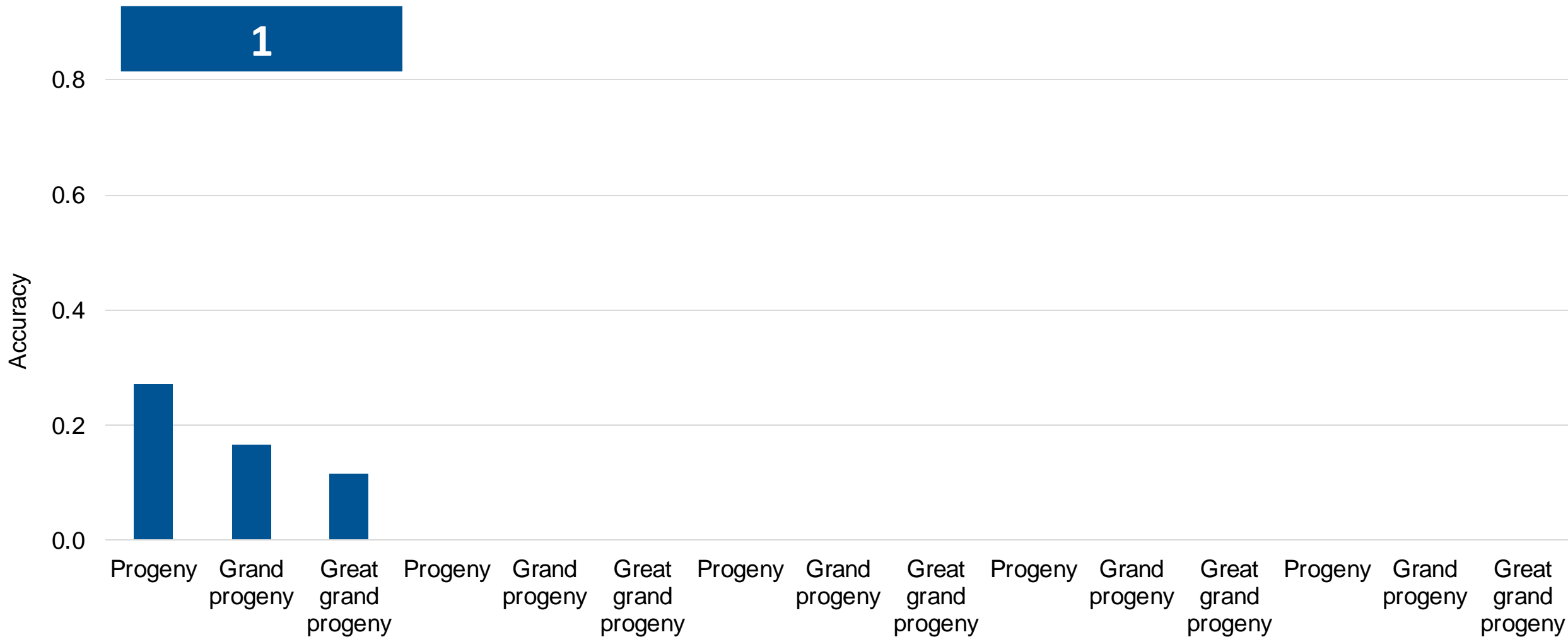


Garcia et al. (2018)

## Single-step Genomic BLUP

### BLUP-based methods

Growth Trait



Hidalgo et al. (2022)

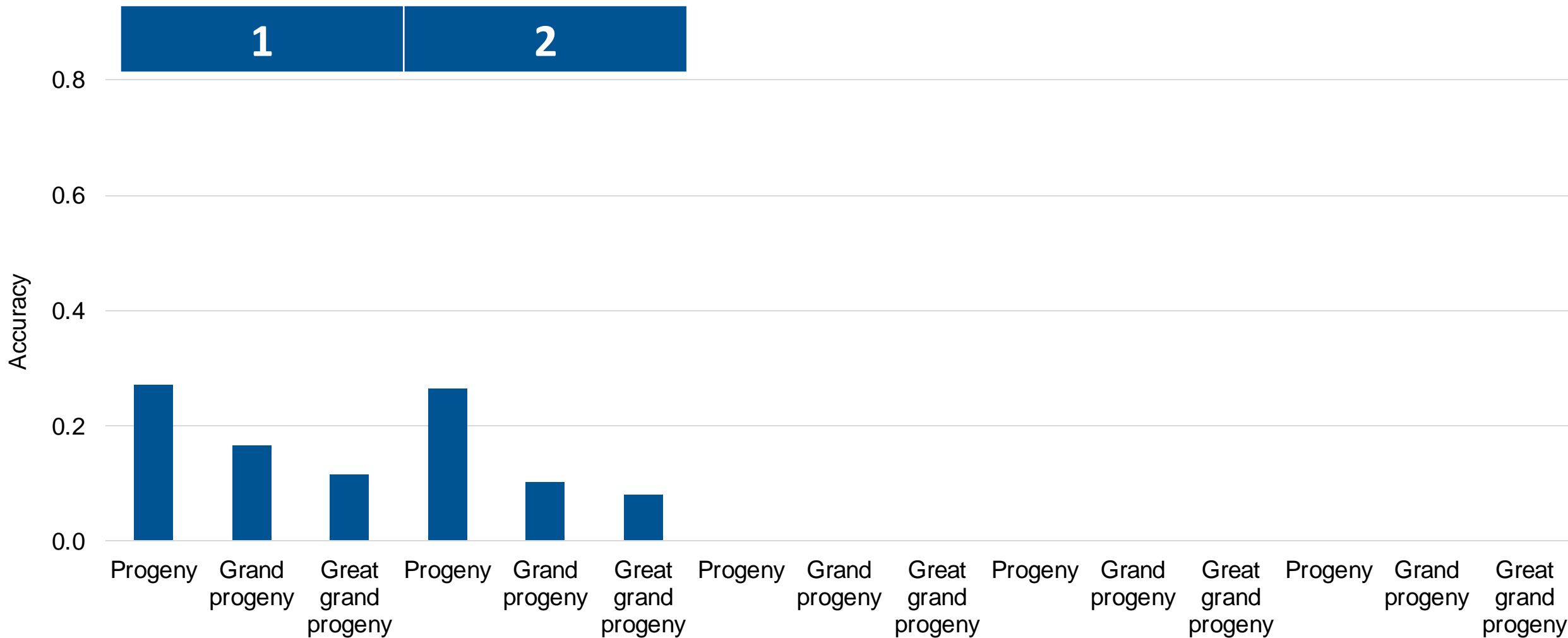
Pedigree + Phenotypes

Pedigree + Phenotypes + Genotypes

## Single-step Genomic BLUP

### BLUP-based methods

Growth Trait



Pedigree + Phenotypes

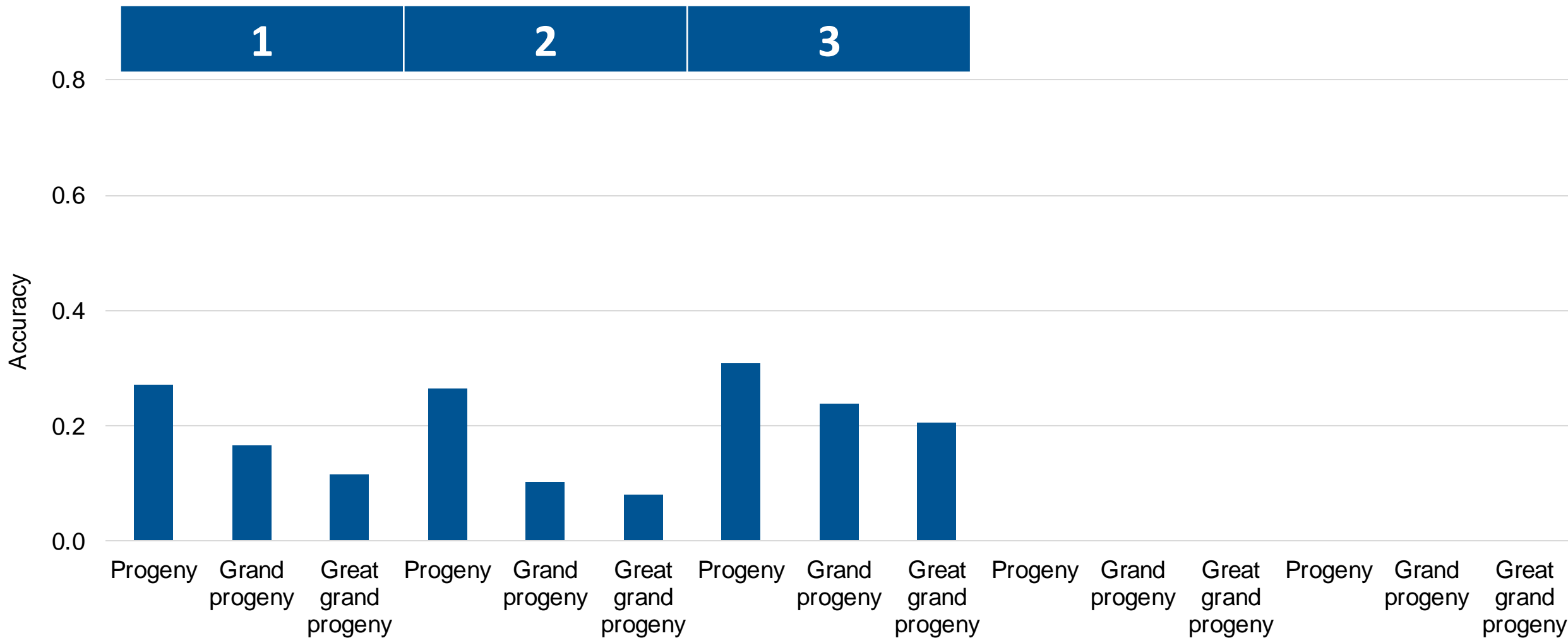
Pedigree + Phenotypes + Genotypes

Hidalgo et al. (2018)

## Single-step Genomic BLUP

### BLUP-based methods

Growth Trait



Pedigree + Phenotypes



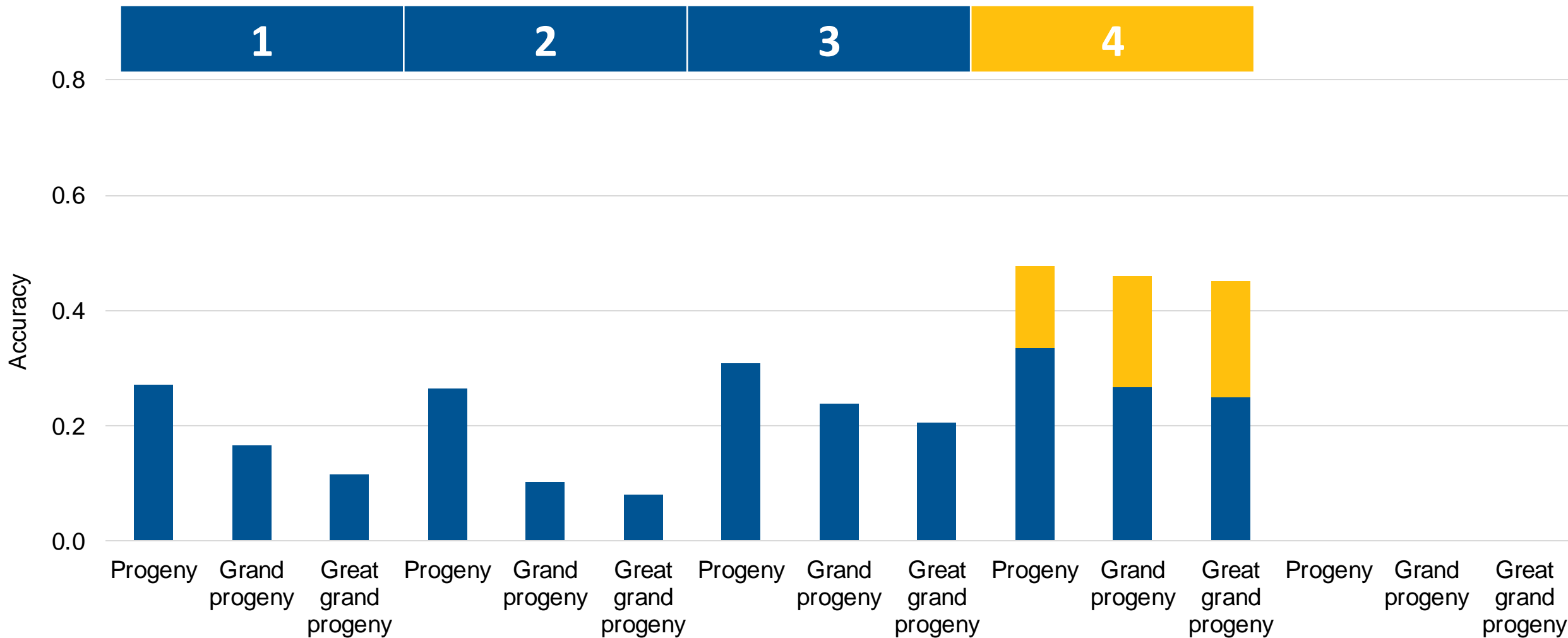
Hidalgo et al. (2018)

Pedigree + Phenotypes + Genotypes

## Single-step Genomic BLUP

### BLUP-based methods

Growth Trait



Hidalgo et al. (2022)

Pedigree + Phenotypes

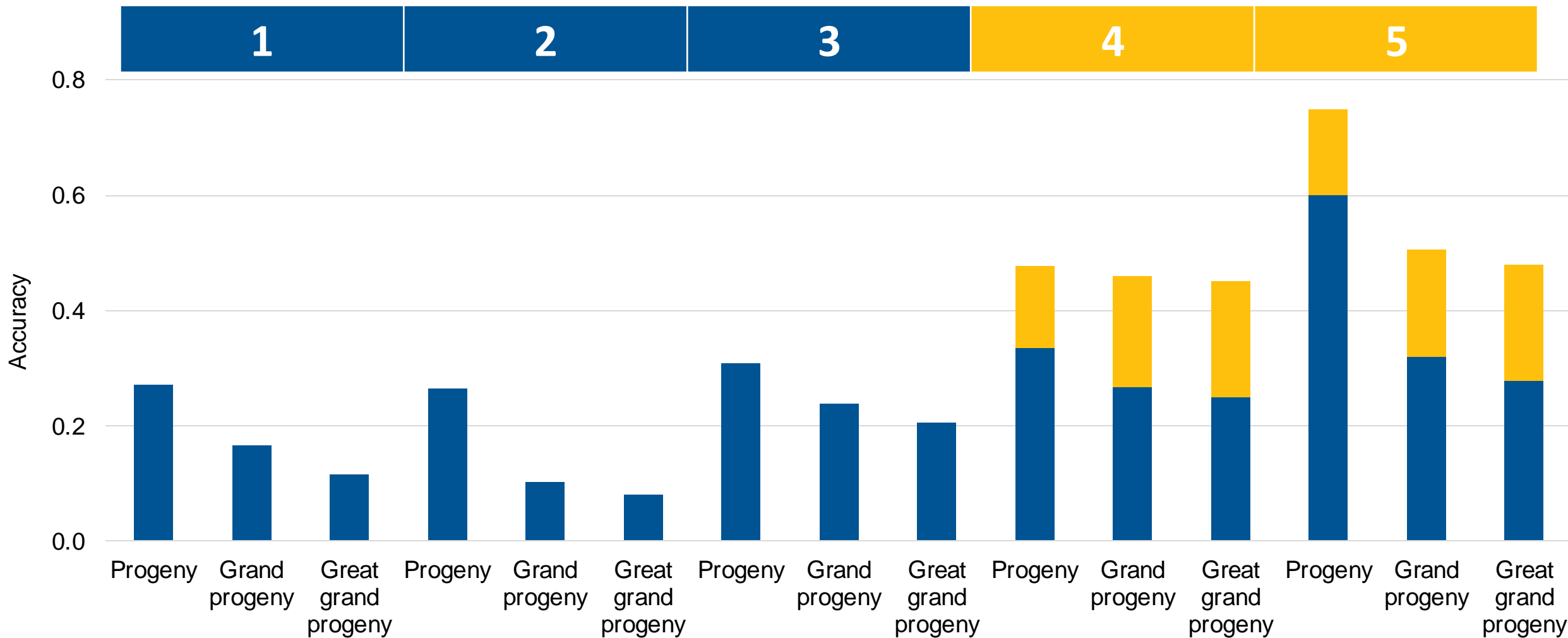
Pedigree + Phenotypes + Genotypes



## Single-step Genomic BLUP

### BLUP-based methods

Growth Trait

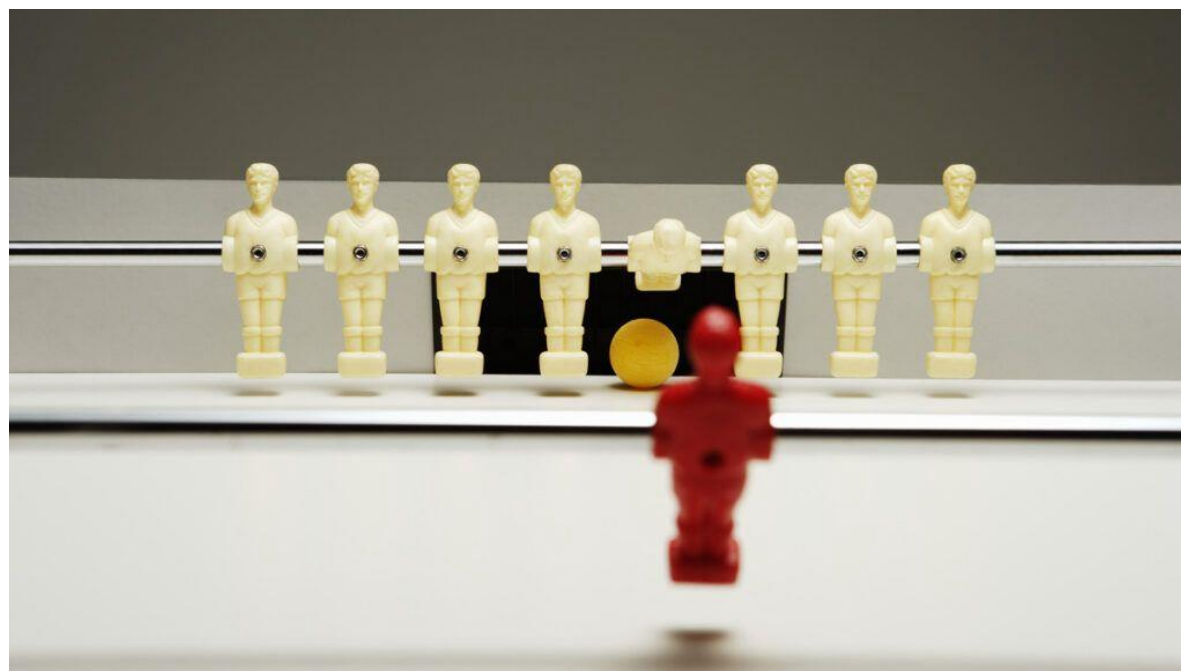


Pedigree + Phenotypes



Hidalgo et al. (2022)

Pedigree + Phenotypes + Genotypes



## How to construct G

# Realized relationship matrix

- Back to 1922, Wright's relationships matrix (A)
- Relationships were conceived as standardized covariances

Wright, S. 1922. Coefficients of inbreeding and relationship. *The American Naturalist* 56:330-338.

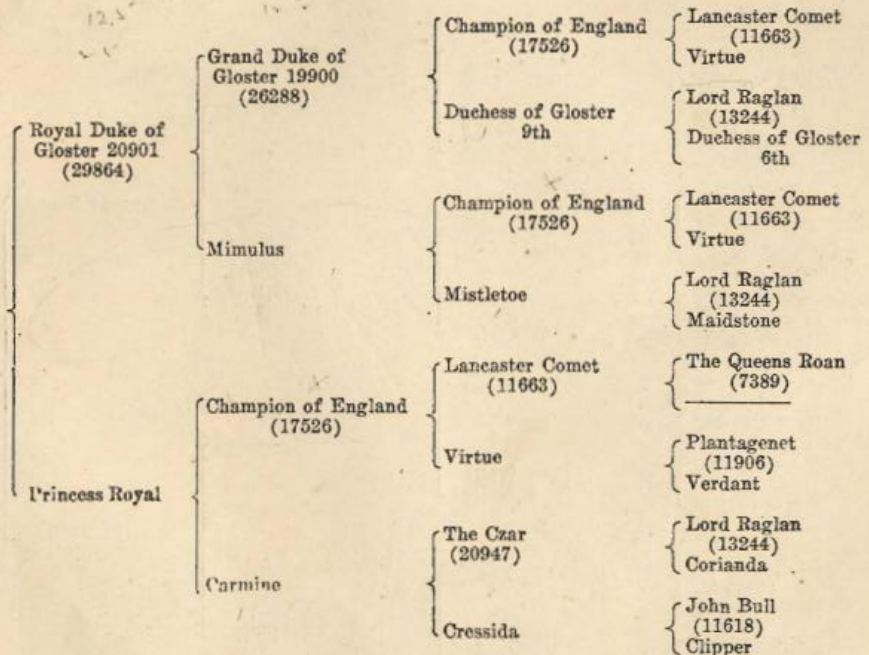
## COEFFICIENTS OF INBREEDING AND RELATIONSHIP

DR. SEWALL WRIGHT

BUREAU OF ANIMAL INDUSTRY, UNITED STATES DEPARTMENT  
OF AGRICULTURE

In the breeding of domestic animals consanguineous matings are frequently made. Occasionally matings are made between very close relatives—sire and daughter, brother and sister, etc.—but as a rule such close inbreeding is avoided and there is instead an attempt to concentrate the blood of some noteworthy individual by what is known as line breeding. No regular system of mating such as might be followed with laboratory animals is practicable as a rule.

Roan Gauntlet  
45276 (35284)



## Realized relationship matrix

- How much DNA do two individuals share looking to DNA?
  - Let gene content be coded as 0, 1, and 2 copies of a reference allele
  - Define  $z_{ij}$  for locus  $i$ , individual  $j$  as the gene content
  - The mean of gene content is twice the allele frequency;  $\bar{z} = 2p$
  - The variance of the gene content;  $\sigma_z^2 = 2p(1 - p)$
- Center  $z_{ij}$  subtracting the mean;  $z_{ij} - 2p_i$
- Scale dividing by the sum of variances =  $\sum 2p_i(1 - p_i)$

ID1	0221200101202211002222121
ID2	2211212121101211212012121
ID3	1212120020202120120122111

$$G = \frac{ZZ'}{\sum 2p_i(1 - p_i)}$$



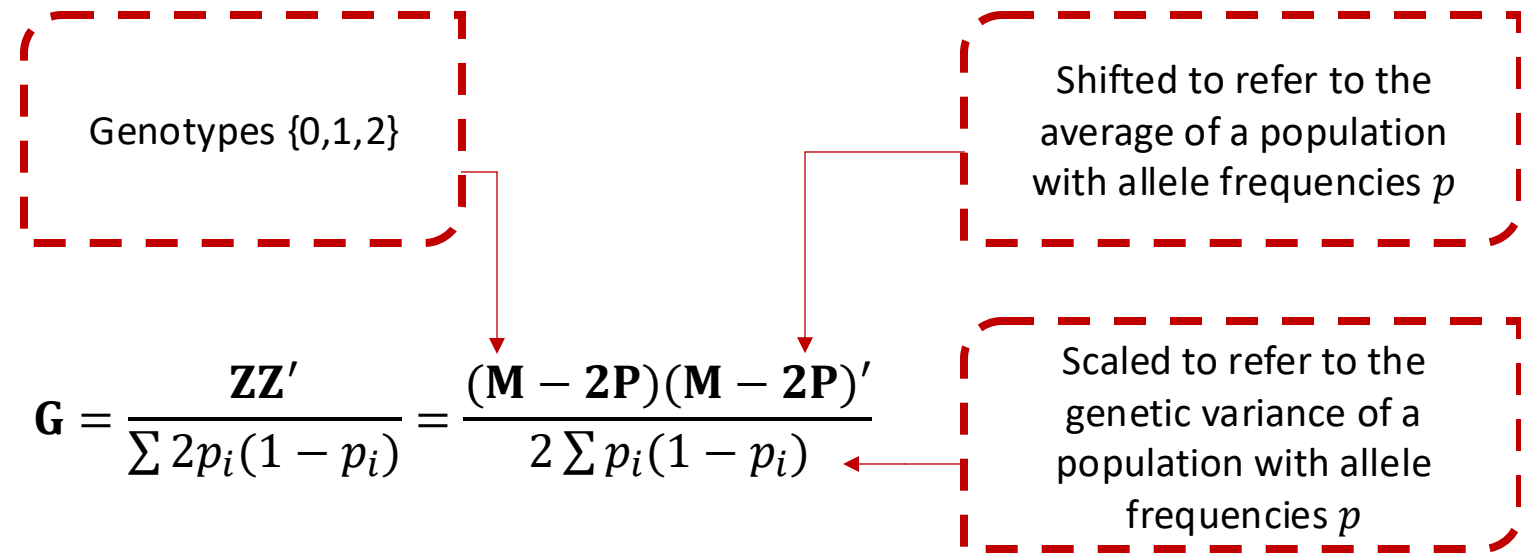
## Realized relationship matrix

Genotypes  $\{0,1,2\}$

$$\mathbf{G} = \frac{\mathbf{Z}\mathbf{Z}'}{\sum 2p_i(1 - p_i)} = \frac{(\mathbf{M} - 2\mathbf{P})(\mathbf{M} - 2\mathbf{P})'}{2 \sum p_i(1 - p_i)}$$

Shifted to refer to the average of a population with allele frequencies  $p$

Scaled to refer to the genetic variance of a population with allele frequencies  $p$





# Realized relationship matrix

- **Tuning** scales  $\mathbf{G}$  to  $\mathbf{A}_{22}$  to refer to the same genetic base
- $p(\mathbf{u}_2) = N(\mathbf{0}, \mathbf{G}\sigma_u^2)$
- If the population is undergoing selection, the mean is not 0
- Different genetic variance in genotyped and ungenotyped animals
- Accounts for the selection, improves accuracy, and reduces bias
  - $(\overline{diag(\mathbf{G})})b + a = \overline{diag(\mathbf{A}_{22})}$
  - $a + b\bar{\mathbf{G}} = \bar{\mathbf{A}}_{22}$
  - $\mathbf{G}_{tun} = a + b\mathbf{G}_0$
- **Blending** avoids singularity; the procedure consists of a weighted sum of  $\mathbf{G}_0$  and a positive-definitive matrix
- Improves convergence
  - $\mathbf{G} = \alpha\mathbf{G}_{tun} + \beta\mathbf{A}_{22}$ 
    - This also assigns part of the genetic variance to pedigrees



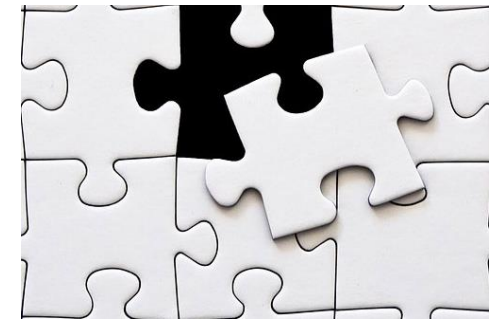
## Importance of inbreeding

# Realized relationship matrix

$$\mathbf{H}^{-1} = \mathbf{A}^{-1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}^{-1} - \mathbf{A}_{22}^{-1} \end{bmatrix}$$

The  $\mathbf{G}$  matrix computed using VanRaden's method considers inbreeding, so  $\mathbf{G}^{-1}$  does. Therefore,  $\mathbf{A}^{-1}$  and  $\mathbf{A}_{22}^{-1}$  should be constructed considering inbreeding to avoid inflation in the estimated breeding values

- Pocrnic et al. (2016)
- 10 generations: 5 males mated 12.5k females
- 138k pedigree | 75k genotyped animals
- Average inbreeding in generation 10 = 0.21
- **No convergence after 5000 iterations**
- Ideal simulated population
- No missing pedigree
- All recent generations were in the pedigree file





## Realized relationship matrix

Computed using  
Henderson-Quaas'  
algorithm, **without**  
**inbreeding**

$\mathbf{A}^{22} < \mathbf{A}_{22}^{-1}$   
Ill conditioned MME  
Inflated GEBV

Computed using Colleau's  
formula, which considers  
inbreeding

$$\mathbf{H}^{-1} = \mathbf{A}^{-1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}^{-1} - \mathbf{A}_{22}^{-1} \end{bmatrix}$$

$$\mathbf{H}^{-1} = \begin{bmatrix} \mathbf{A}^{11} & \mathbf{A}^{12} \\ \mathbf{A}^{21} & \mathbf{A}^{22} \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}^{-1} - \mathbf{A}_{22}^{-1} \end{bmatrix}$$



Computed using VanRaden's  
formula, which considers  
inbreeding

## Importance of inbreeding

# Realized relationship matrix

Computed using  
Henderson-Quaas'  
algorithm, with  
inbreeding

Computed using Colleau's  
formula, which considers  
inbreeding

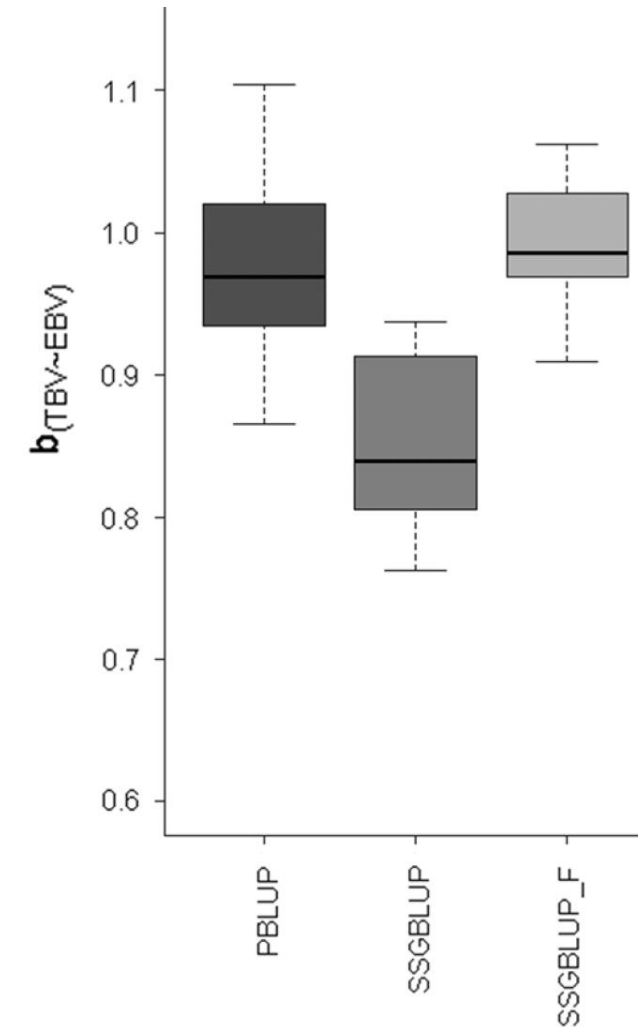
$$\mathbf{H}^{-1} = \mathbf{A}^{-1} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}^{-1} - \mathbf{A}_{22}^{-1} \end{bmatrix}$$

Computed using VanRaden's  
formula, which considers  
inbreeding



## Realized relationship matrix

- Garcia-Baccino et al. (2017)
- 29k pedigree | 5.3k genotyped animals
- PBLUP vs. ssGBLUP vs. ssGBLUP\_inbreeding (F)
- Inflated GEBV with ssGBLUP
- No inflation with inbreeding



# Realized relationship matrix

Inbreeding is also important in the estimation of accuracies

$$Accuracy_i = \sqrt{1 - \left[ \frac{PEV_i}{\sigma_u^2(1 + F_i)} \right]}$$

Received: 22 August 2019 | Revised: 10 December 2019 | Accepted: 11 January 2020  
DOI: 10.1111/jbg.12470

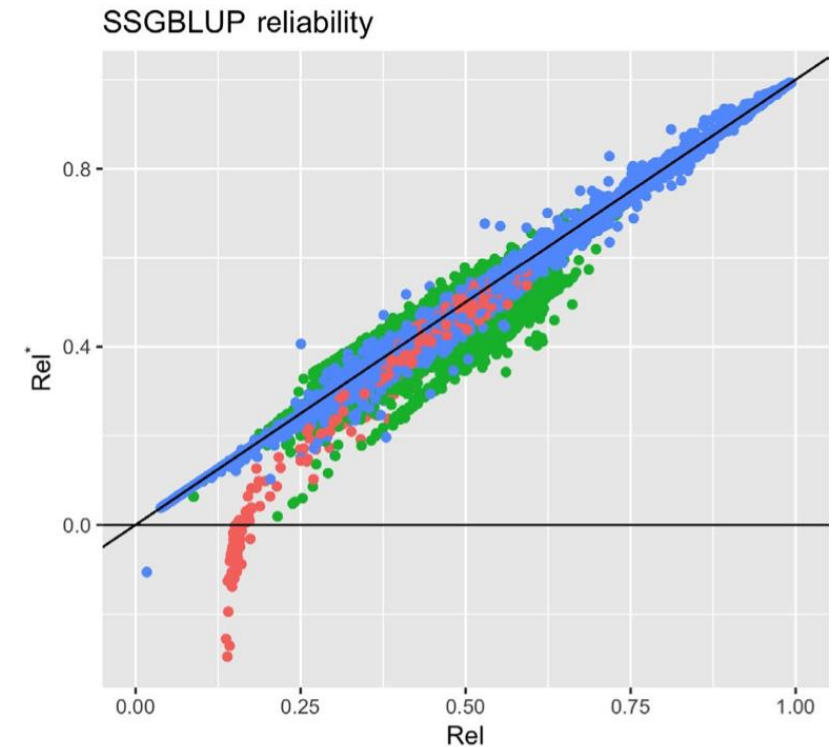


## ORIGINAL ARTICLE

Journal of Animal Breeding and Genetics | WILEY

### Effects of ignoring inbreeding in model-based accuracy for BLUP and SSGBLUP

Ignacio Aguilar<sup>1</sup> | Eduardo N. Fernandez<sup>2</sup> | Agustin Blasco<sup>3</sup> | Olga Ravagnolo<sup>1</sup> | Andres Legarra<sup>4</sup>



# Decomposition of EBV and GEBV

EBV

$$\{\mathbf{W}'\mathbf{W} + \mathbf{A}^{-1}\lambda\}\hat{\mathbf{u}} = \mathbf{W}'\mathbf{y}$$

$$u_i = w_1 PA_i + w_2 YD_i + w_3 PC_i$$

Parent Average	Yield Deviation	Progeny Contribution
-------------------	--------------------	-------------------------

GEBV

$$\left\{ \mathbf{W}'\mathbf{W} + \mathbf{A}^{-1}\lambda + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{G}^{-1} - \mathbf{A}_{22}^{-1} \end{bmatrix} \lambda \right\} \hat{\mathbf{u}} = \mathbf{W}'\mathbf{y}$$

$$u_i = w_1 PA_i + w_2 YD_i + w_3 PC_i + (w_{4_1} DGV_i - w_{4_2} PP_i)$$

Parent Average	Yield Deviation	Progeny Contribution	Direct Genomic Value	Pedigree Prediction
-------------------	--------------------	-------------------------	-------------------------	------------------------

For young animals

$$u_i = w_1 PA_i + (w_{4_1} DGV_i - w_{4_2} PP_i)$$

With many genotypes

$$u_i \approx w_{4_1} DGV_i$$



## Decomposition of EBV and GEBV

- For young animals  $u_i = w_1 PA_i + (w_{4_1} DGV_i - w_{4_2} PP_i)$

$$u_i = \frac{\frac{2}{1-F_i}}{\frac{2}{1-F_i} + g^{ii} - a_{22}^{ii}} PA_i + \left( \frac{g^{ii}}{\frac{2}{1-F_i} + g^{ii} - a_{22}^{ii}} DGV_i - \frac{a^{ii}}{\frac{2}{1-F_i} + g^{ii} - a_{22}^{ii}} PP_i \right)$$

- Ignoring F

$$u_i = \frac{2}{2 + g^{ii} - a_{22}^{ii}} PA_i + \left( \frac{g^{ii}}{2 + g^{ii} - a_{22}^{ii}} DGV_i - \frac{a^{ii}}{2 + g^{ii} - a_{22}^{ii}} PP_i \right)$$

- Inbreeding increases the denominator
- GEBV is smaller
- Inflation is reduced



## Estimating Variance Components

We require VC or at least some function of them

EM-REML

$$\mathbf{y} = \mathbf{X}\mathbf{b} + \mathbf{Z}\mathbf{u} + \mathbf{e}$$

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{A}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix} \quad \lambda = \frac{\hat{\sigma}_e^2}{\hat{\sigma}_u^2}$$

1. Set initial variance components
2. Compute  $\hat{\mathbf{b}}$  and  $\hat{\mathbf{u}}$  solving the MME
3. Update variance components

$$\hat{\sigma}_u^2 = \frac{\hat{\mathbf{u}}'\mathbf{A}^{-1}\hat{\mathbf{u}} + tr(\mathbf{A}^{-1}\mathbf{C}^{uu})\hat{\sigma}_e^2}{N}$$

← Inverse of LHS for individual effect

← Number of individuals, rank of  $\mathbf{A}$

$$\hat{\sigma}_e^2 = \frac{\mathbf{y}'(\mathbf{y} - \mathbf{X}\hat{\mathbf{b}} - \mathbf{Z}\hat{\mathbf{u}})}{N - rank(\mathbf{X})}$$

4. Go to 1 or stop if variance components do not change anymore

Patterson and Thompson (1971)

Dempster et al. (1977)

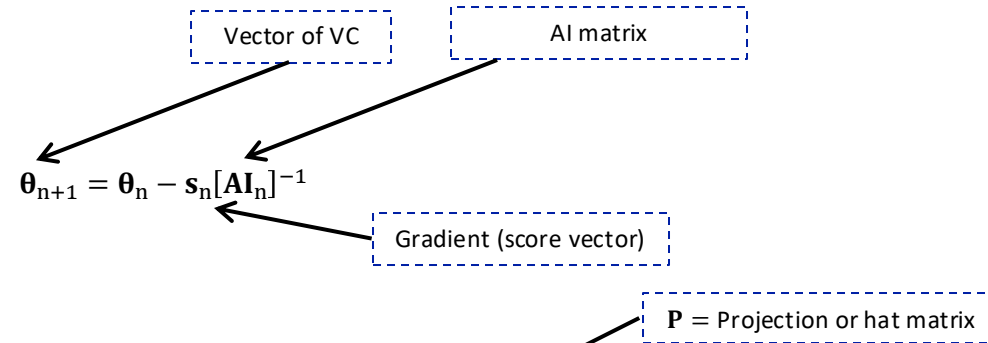




# Estimating Variance Components

AI-REML

AI- algorithm uses this matrix as Hessian



$$AI = \begin{bmatrix} \frac{1}{2} \left\{ \frac{(y - X\hat{b} - Z\hat{u})' P (y - X\hat{b} - Z\hat{u})}{(\hat{\sigma}_e^2)^2} \right\} & \frac{1}{2} \left\{ \frac{u' Z' P (y - X\hat{b} - Z\hat{u})}{\hat{\sigma}_e^2 (\hat{\sigma}_u^2)^2} \right\} \\ \frac{1}{2} \left\{ \frac{(y - X\hat{b} - Z\hat{u})' P Z u}{\hat{\sigma}_e^2 (\hat{\sigma}_u^2)^2} \right\} & \frac{1}{2} \left\{ \frac{u' Z' P Z u}{(\hat{\sigma}_u^2)^2} \right\} \end{bmatrix}$$

Gradient

$$s = \begin{bmatrix} \frac{1}{2} \left\{ \frac{(y - X\hat{b} - Z\hat{u})' (y - X\hat{b} - Z\hat{u})}{(\hat{\sigma}_e^2)^2} - \frac{(n - p - q)}{(\hat{\sigma}_e^2)^2} - \frac{tr(A^{-1} C^{uu})}{\hat{\sigma}_u^2} \right\} \\ \frac{1}{2} \left\{ \frac{\hat{u}' A^{-1} \hat{u}}{(\hat{\sigma}_u^2)^2} - \frac{q}{\hat{\sigma}_u^2} + tr(A^{-1} C^{uu}) \frac{\hat{\sigma}_e^2}{(\hat{\sigma}_u^2)^2} \right\} \end{bmatrix}$$



## Estimating Variance Components

### EM-REML

- Simple equations
  - More complex in multiple-trait models
- Very slow convergence
- Computationally demanding ( $C^{uu}$ )

### AI-REML

- Faster than EM-REML
  - Fewer iterations
- Provides estimation of standard errors
- For complex models and poor starting values
  - Slow convergence
  - Estimates out of the parameter space
- Initial rounds with EM-REML may help
- Computationally demanding ( $C^{uu}$ )

$$\begin{bmatrix} \mathbf{X}'\mathbf{X} & \mathbf{X}'\mathbf{W} \\ \mathbf{W}'\mathbf{X} & \mathbf{W}'\mathbf{W} + \mathbf{A}^{-1}\lambda \end{bmatrix} \begin{bmatrix} \hat{\mathbf{b}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{y} \\ \mathbf{W}'\mathbf{y} \end{bmatrix}$$



# gibbsf90+

- `gibbs1f90`: stores single trait matrices once – fast for multi-trait models
- `gibbs2f90`: `gibbs1f90` with joint sampling of correlated effects – Maternal effects and RRM
- `gibbs3f90`: `gibbs2f90` with heterogeneous residual variance
- `thrgibbs1f90`: for linear-threshold models
- `thrgibbs3f90`: `thrgibbs1f90` with heterogeneous residual variance

## Variance Components Estimation

### Mixed Model Equations Solver

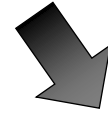
$$\begin{bmatrix} \mathbf{X}'\mathbf{R}^{-1}\mathbf{X} & \mathbf{X}'\mathbf{R}^{-1}\mathbf{W} \\ \mathbf{W}'\mathbf{R}^{-1}\mathbf{X} & \mathbf{W}'\mathbf{R}^{-1}\mathbf{W} + \mathbf{A}^{-1} \otimes \mathbf{G}_0^{-1} \end{bmatrix} \begin{bmatrix} \hat{\boldsymbol{\beta}} \\ \hat{\mathbf{u}} \end{bmatrix} = \begin{bmatrix} \mathbf{X}'\mathbf{R}^{-1}\mathbf{y} \\ \mathbf{W}'\mathbf{R}^{-1}\mathbf{y} \end{bmatrix}$$

# gibbsf90+



Linear

Default



Threshold (-Linear)

OPTION cat 0 2 5

- Categories renumbered from **1**
- Missing records is only **0**

# gibbsf90+

## Bayes Theorem

$$p(\theta|y) = p(y|\theta) p(\theta)$$

The diagram illustrates the components of Bayes' Theorem. It shows the equation  $p(\theta|y) = p(y|\theta) p(\theta)$ . Three red arrows point from the terms in the equation to their definitions: one from  $p(y|\theta)$  to 'Likelihood function', one from  $p(\theta)$  to 'prior probability of unknown  $\theta$ ', and one from  $p(\theta|y)$  to 'posterior probability of unknown  $\theta$  with known  $y$ '. The definition for the likelihood function also includes a sub-explanation: 'indicates how likely the observations are from a distribution (with particular parameters)'.

Likelihood function  
indicates how likely the observations are from a distribution  
(with particular parameters)

prior probability of unknown  $\theta$

posterior probability of unknown  $\theta$  with known  $y$

- Basic idea of Gibbs Sampling:
- Numerical method to draw samples from a posterior distribution (not always explicitly available)
- Draw samples = generate random numbers following a distribution
- The results are random numbers (not theoretical formulas)
- The posterior distribution will be drawn based on the numerical values (like a histogram)

# gibbsf90+

## Ingredients for Gibbs sampling

- 1) Theoretical derivation: conditional posterior distribution for each unknown parameter
- 2) Software: a random number generator for a particular distribution

```
# Basic Gibbs sampling for mu (normal) and sigma2 (inverted chi-square)
y <- c(14,16,18)
N <- length(y)
n.samples <- 100
mu <- rep(0,n.samples)
sigma2 <- rep(0,n.samples)

# initial value
mu[1] <- 0
sigma2[1] <- 10

# sampling
for(i in 2:n.samples){
  mu[i] <- rnorm(1, mean=mean(y), sd=sqrt(sigma2[i-1]/N)) # using the most recent sigma2
  df <- N-2
  S <- sum((y-mu[i])^2)
  sigma2[i] <- rinvchisq(1, df=df, scale=S) # using the most recent mu
}
```

# gibbsf90+

- Name of parameter file?

`gibbs1.par`

- Number of samples and length of burn-in?

`samples=10,000 to 100,000; burn-in=0`

- Give n to store every n-th sample?

`10`

- `gibbsf90+ parfile.par --samples i --burnin j --interval k`

# gibbsf90+

- Procedure
  - Run `gibbsf90+` to estimate variance components
  - Run `postgibbsf90` to process the samples and check convergence
  - Run `gibbsf90+` with new variance components to compute EBV (2k to 10k samples)

```
OPTION fixed_var mean X
```



Number of the  
animal effect



# postgibbsf90

- Basic idea of post-Gibbs analysis:
- Summarize and visualize the samples drawn by gibbsf90+
- Confirm if the chain converged
- Find the most probable value = posterior mode as a “point estimate”
- Find the reliability of the estimates = the highest posterior density as a “confidence interval”

# postgibbsf90

- Name of parameter file?  
gibbs1.par
- Burn-in?  
0
- Give n to store every n-th sample? (1 means read all samples)  
10
- input files  
gibbs\_samples, fort.99
- output files
  - "postgibbs\_samples"  
all Gibbs samples for additional post analyses
  - "postmean"  
posterior means
  - "postsd"  
posterior standard deviations
  - "postout"

# postgibbsf90

at least > 10 is recommended  
> 30 may be better

					*****	Monte	Carlo	Error by	Time Series	*****		
Pos.	eff1	eff2	trt1	trt2	MCE	Mean	HPD	Interval (95%)	Effective sample size	Median	Mode	Independent chain size
1	4	4	1	1	1.362E-02	0.9889	0.7788	1.215	70.4	0.9844	0.9861	18
2	4	4	1	2	1.288E-02	1.006	0.777	1.219	84.1	1.006	0.952	18
3	4	4	2	2	1.847E-02	1.66	1.347	1.987	80.3	1.652	1.579	25
4	0	0	1	1	9.530E-03	24.47	24.07	24.84	425.6	24.47	24.53	2
5	0	0	1	2	8.253E-03	11.84	11.54	12.18	395.8	11.83	11.82	2
6	0	0	2	2	1.233E-02	30.1	29.65	30.58	387.8	30.09	29.97	5

\*\*\*\*\* p<sub>i</sub> Lower and upper bounds of Mean  $\pm$  1.96PSD  
ratio between first half and second half of the samples ; should be < 1.0

Pos.	eff1	eff2	trt1	trt2	PSD	Mean	PSD		Geweke	Autocorrelations			Independent
							Interval (95%)		diagnostic	lag: 1	10	50	# batches
1	4	4	1	1	0.1144	0.9889	0.7648	1.213	-0.02	0.853	0.188	0.049	50
2	4	4	1	2	0.1182	1.006	0.7742	1.237	-0.11	0.828	0.111	-0.066	50
3	4	4	2	2	0.1656	1.66	1.335	1.984	0.06	0.828	0.108	-0.021	36
4	0	0	1	1	0.1967	24.47	24.09	24.86	-0.01	0.034	0.029	-0.062	450
5	0	0	1	2	0.1643	11.84	11.51	12.16	0.03	0.032	-0.006	-0.016	450
6	0	0	2	2	0.2429	30.1	29.62	30.57	-0.02	0.07	-0.014	0.037	180

# postgibbsf90

```
Choose a graph for samples (= 1) or histogram (= 2); or exit (= 0)
```

```
1
```

```
positions
```

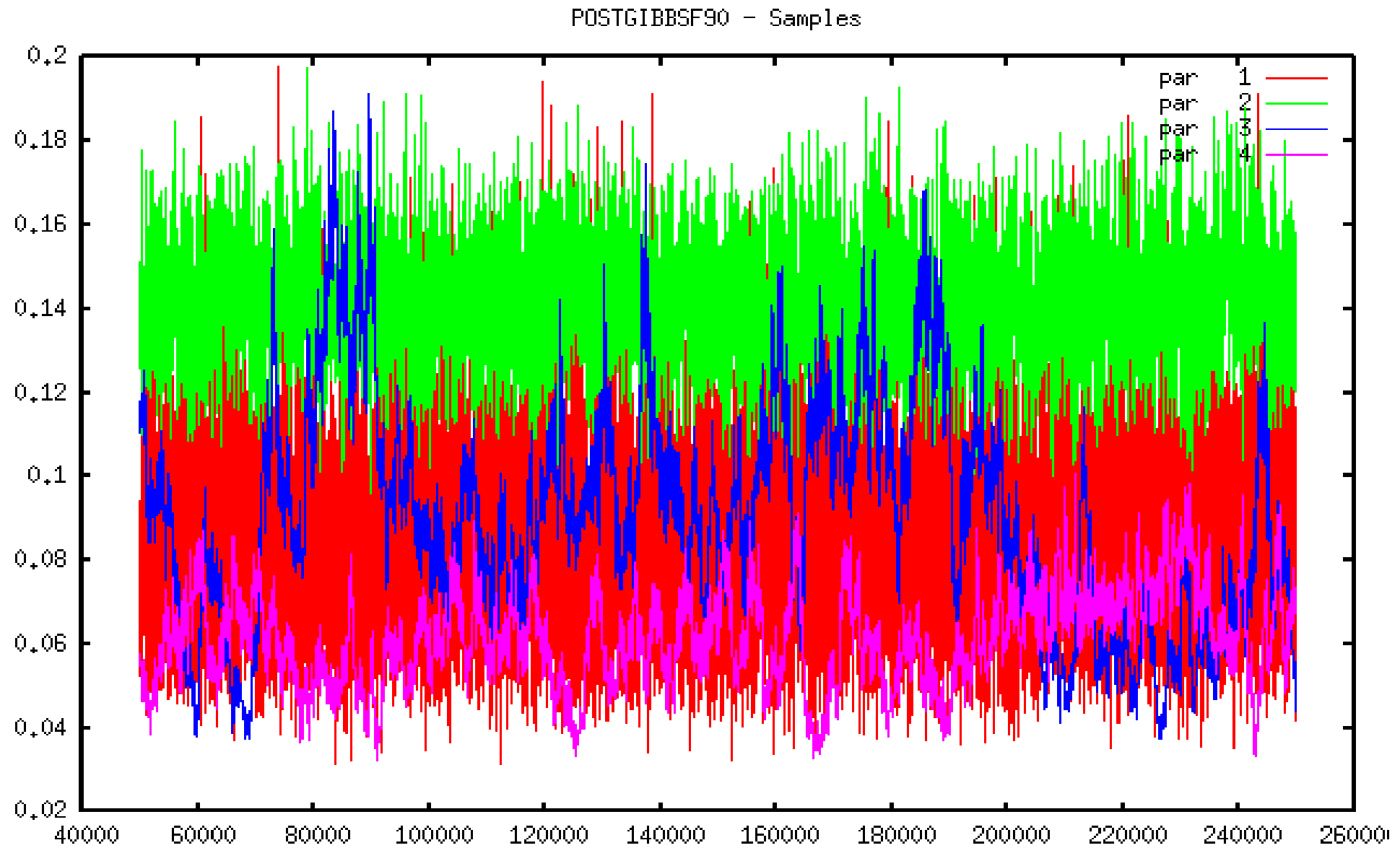
```
1 2 3 # choose from the position numbers 1 through 6
```

```
If the graph is stable (not increasing or decreasing), the convergence is met.  
All samples before that point should be discarded as burn-in.
```

```
print = 1; other graphs = 2; or stop = 0
```

```
2
```

# postgibbsf90



# postgibbsf90

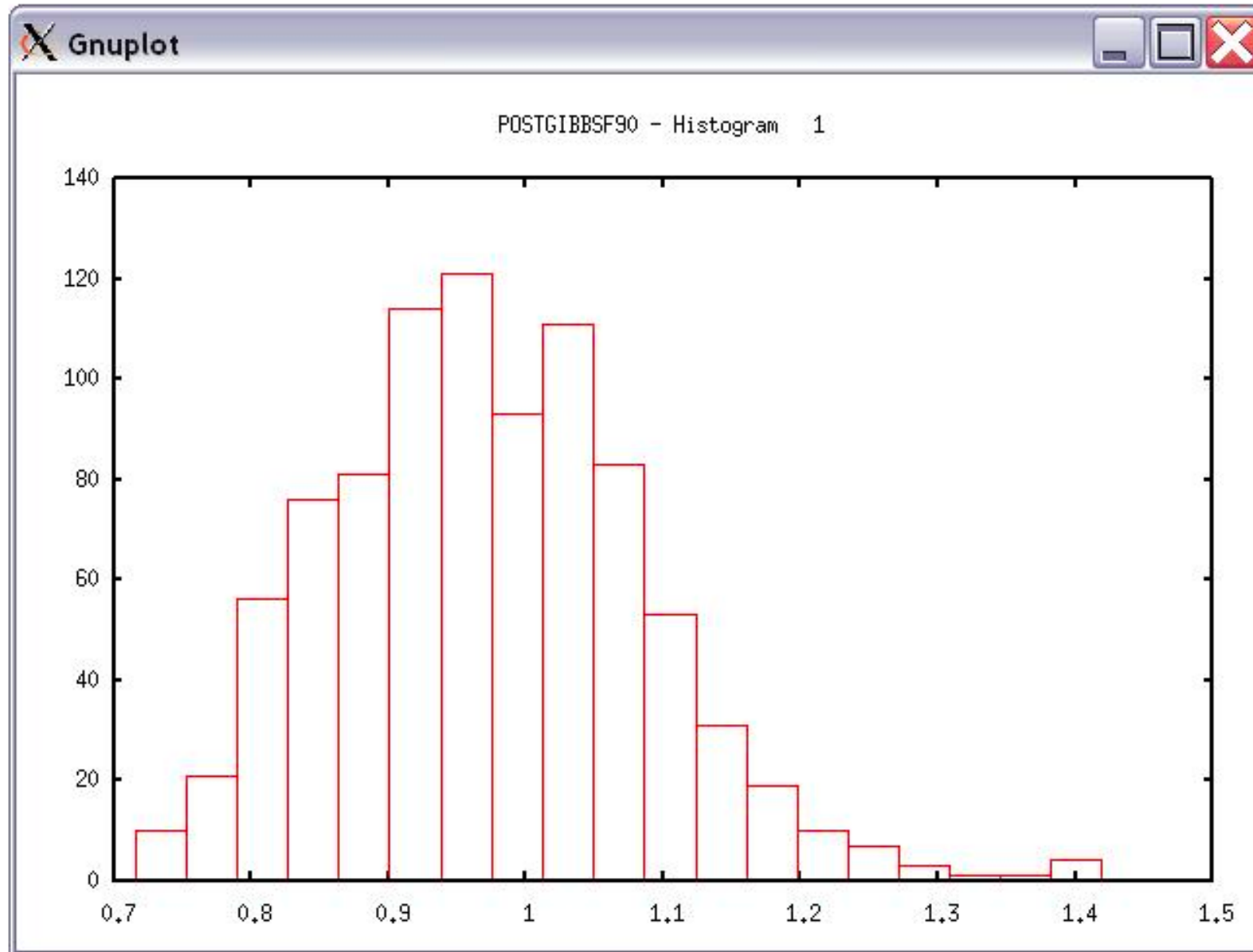
```
Choose a graph for samples (= 1) or histogram (= 2); or exit (= 0)
```

```
2
```

```
Type position and # bins
```

```
1 20
```

# postgibbsf90



# Common problems for BLUPF90 family

- Wrong position or formats for observation and effects
- Misspelling of Keywords
  - Program may stop
- (Co)variance matrices not symmetric, not positive definite
  - Program may not stop
- Large numbers (e.g., 305-day milk yield 10,000 kg)
  - Scale down i.e.,  $10,000 / 1,000 = 10$



# General output from BLUPF90 family

- Output printed on the screen is not saved to any file!
- Should use redirection or pipes to store output

## **renumf90**

```
renumf90 renum.par | tee renum.log
```

## **blupf90+**

```
blupf90+ renumf90.par | tee blup.log
```

## **gibbsf90+**

```
gibbsf90+ exmr99s1 --samples 1000 --burnin 0 --interval 1 | tee gibbs.log
```

# Run in background + Save output

```
$vi gibbs.sh
#type the following commands inside gibbs.sh
    gibbsf90+ <<AA > gibbs.log
    renf90.par
    1000 0
    10
    AA
#save and exit
$bash gibbs.sh & #can replace bash with sh
```

```
$vi bp.sh
#type the following commands inside bp.sh
    blupf90+ <<AA > blup.log
    renf90.par
    AA
#save and exit
$bash bp.sh & #can replace bash by sh
```

## Estimating Variance Components

### MME Solver

blupf90+

#### Default

- Preconditioner Conjugate Gradient (PCG)
  - Default Iterative method (fast)
- Successive over-relaxation (SOR)
  - An iterative method based on Gauss-Seidel
- Direct solution using sparse Cholesky factorization
  - FSPAK or YAMS (greater memory requirements)

### VC Estimation

- AI-REML:

OPTION method VCE

- EM-REML:

OPTION method VCE

OPTION EM-REML xx

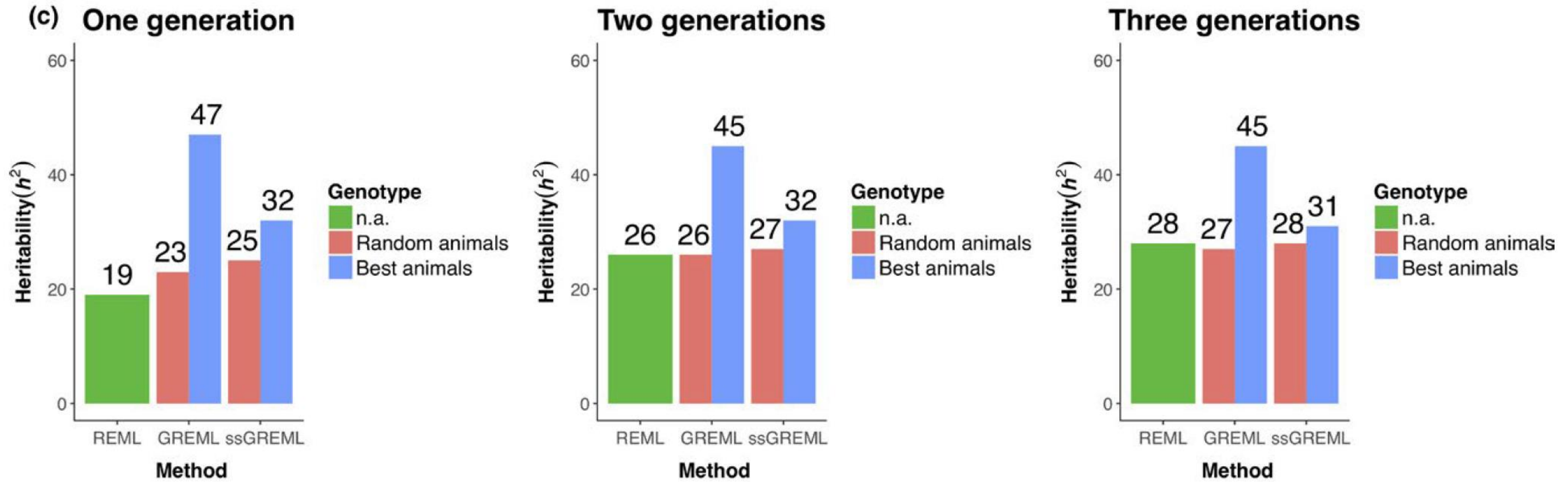
→ \_ (empty for pure EM)  
# of EM rounds  
ai (until convergence)



## *Difference in estimates depending on population structure*

# Estimating Variance Components

- In practice, it is hard to have base allele frequencies
- SSGREML was less affected by selective or limited genotyping



- Estimated heritability = 30%



Cesarani et al. (2018)

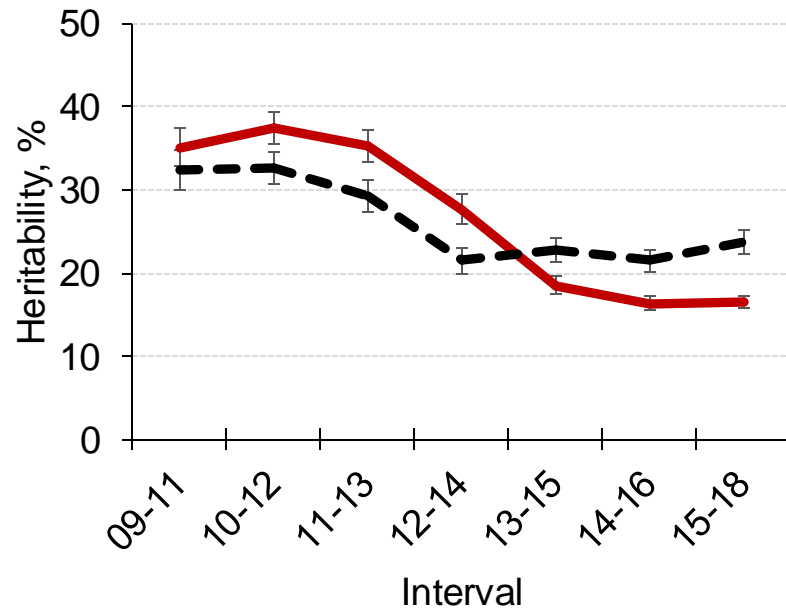
*Difference in estimates depending on population structure*

## Estimating Variance Components

--- Pedigree-based analysis

— Genomic-based analysis

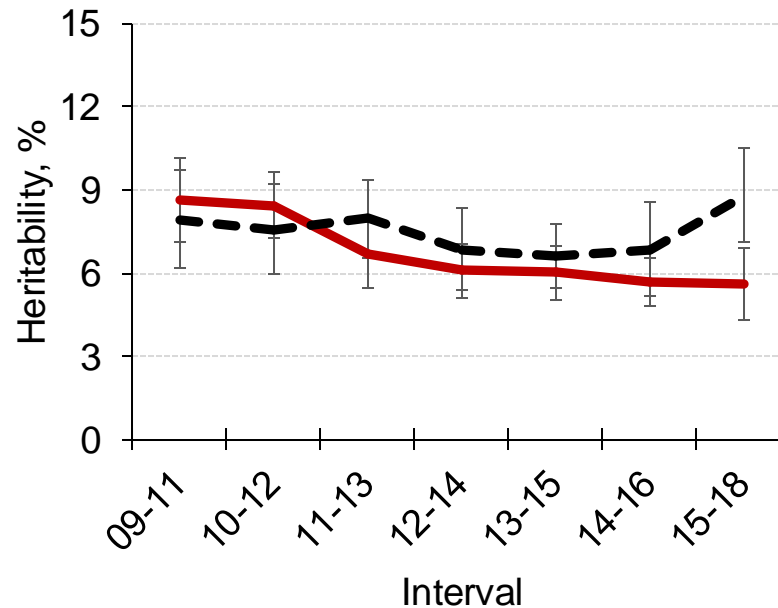
GT



35.1 to 16.5%

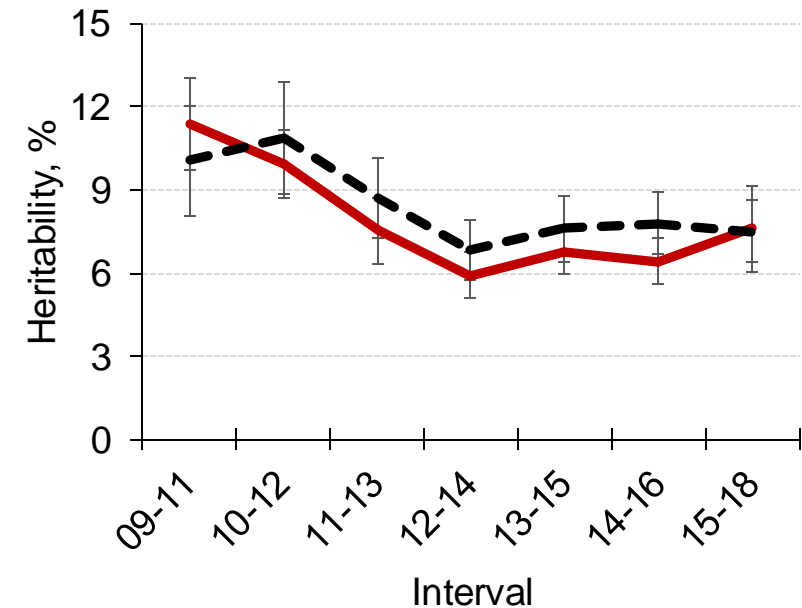
A reduction of ~ 50%

FT1



8.6 to 5.6%

FT2



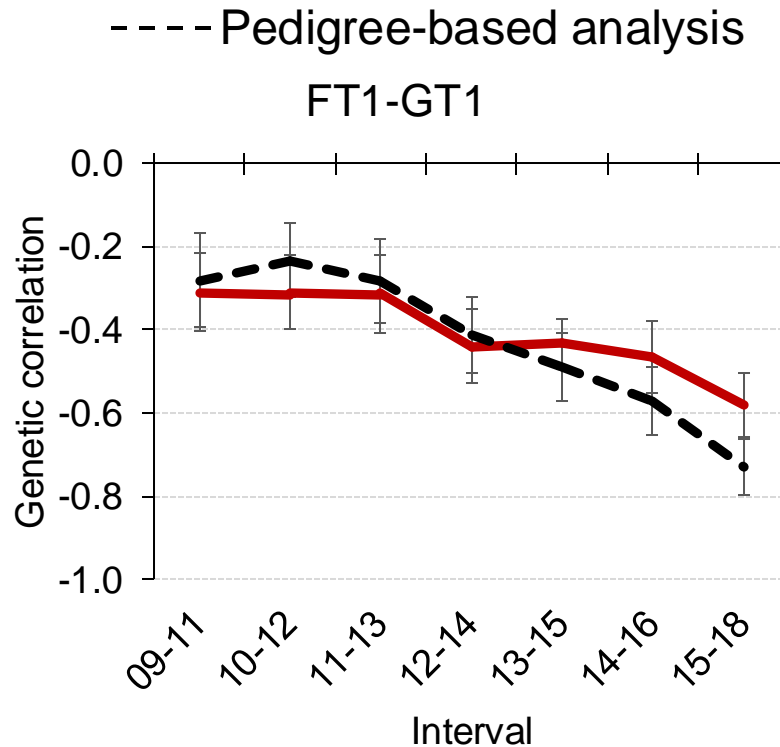
11.4 to 7.6%

~ 20%

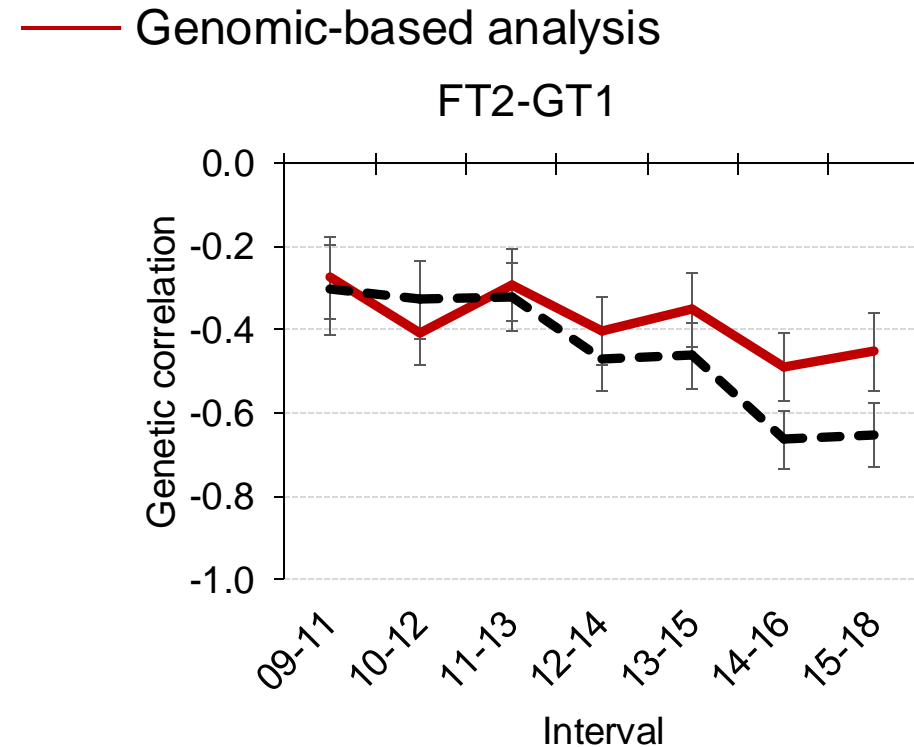


## *Difference in estimates depending on population structure*

# Estimating Variance Components



-0.31 to -0.58



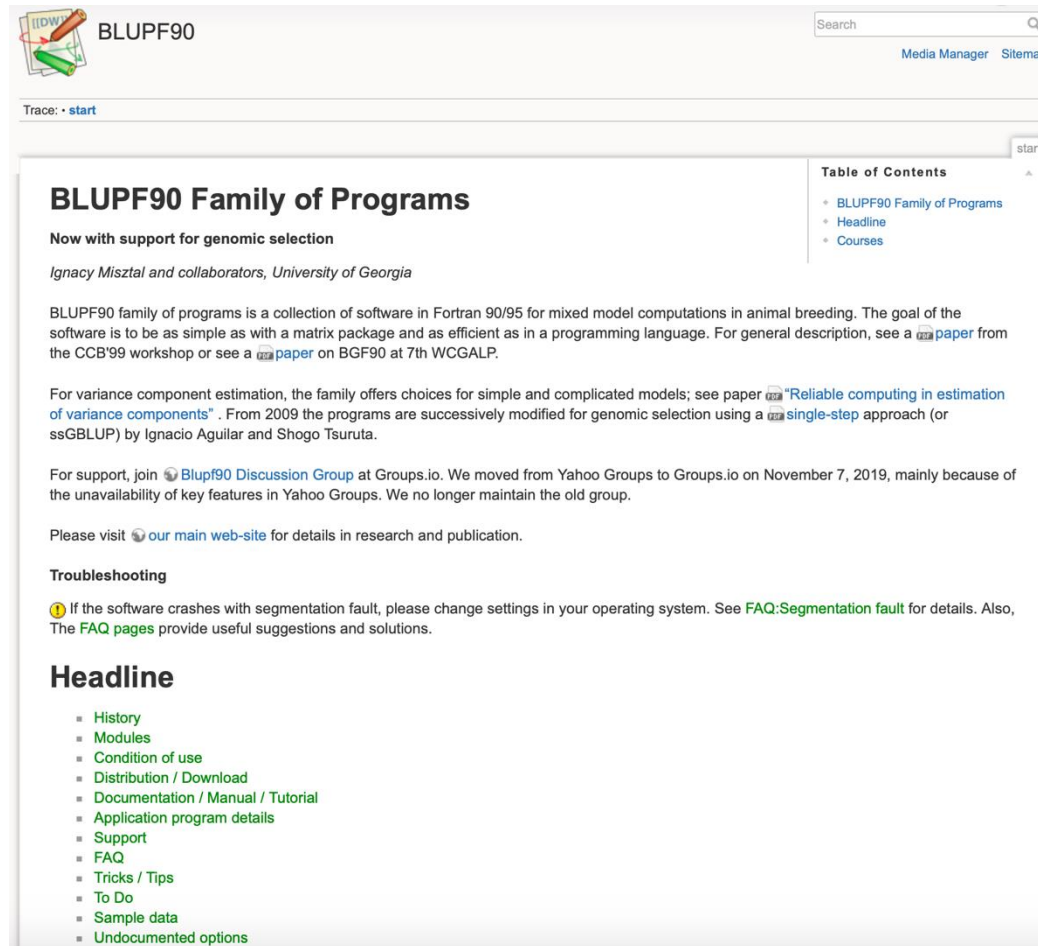
-0.27 to -0.45

- These changes need to be considered in the breeding program



# Blupf90 programs

## Practice



The screenshot shows the BLUPF90 website. At the top left is the BLUPF90 logo, which consists of a stylized 'B' and 'F' with a pencil and a ruler. To the right of the logo is the text 'BLUPF90'. Further right is a search bar with the word 'Search' and a magnifying glass icon. Below the search bar are links for 'Media Manager' and 'Sitemap'. Below these links is a breadcrumb trail that says 'Trace: • start'. The main content area is titled 'BLUPF90 Family of Programs'. Below this title is a sub-header 'Now with support for genomic selection' and a line of text 'Ignacy Misztal and collaborators, University of Georgia'. The main text describes the BLUPF90 family of programs as a collection of software in Fortran 90/95 for mixed model computations in animal breeding. It mentions the goal of the software is to be as simple as with a matrix package and as efficient as in a programming language. It also mentions a paper from the CCB'99 workshop and a paper on BGF90 at 7th WCGALP. There are links to a paper on 'Reliable computing in estimation of variance components' and a 'single-step' approach (or ssGBLUP) by Ignacio Aguilar and Shogo Tsuruta. It also mentions a 'Bluef90 Discussion Group' at Groups.io and a 'main web-site' for details in research and publication. There is a 'Troubleshooting' section with a warning icon and text about segmentation faults. At the bottom is a 'Headline' section with a list of links: History, Modules, Condition of use, Distribution / Download, Documentation / Manual / Tutorial, Application program details, Support, FAQ, Tricks / Tips, To Do, Sample data, and Undocumented options. On the right side of the main content area is a 'Table of Contents' with links to 'BLUPF90 Family of Programs', 'Headline', and 'Courses'.

- Collection of software
- Fortran  $\geq 90$
- Computations in AB & G
- Since 1997 by Ignacy Misztal
- Several developers + collaborators
- Simple, efficient, and comprehensive
- Very general models

<https://nce.ads.uga.edu>

<https://nce.ads.uga.edu/software/>



## Practice



Ignacy  
Misztal



Shogo  
Tsuruta



Andres  
Legarra



Ignacio  
Aguilar



Yutaka  
Masuda



Matias  
Bermann

- + Several contributors
- Research turns into code





## Practice

- **breedR** is FOSS. Licensed [GPL-3](#)
  - `RShowDoc('LICENSE', package = 'breedR')`
- You can **use** and **distribute breedR** for any purpose
- You can **modify** it to suit your needs
  - we encourage to!
  - please consider contributing your improvements
  - you can **distribute** your modified version under the GPL
- However, **breedR** makes (intensive) use of the [BLUPF90](#) suite of Fortran programs

```
res <- remlf90(fixed = phe_X ~ 1,  
              random = ~ gg,  
              data = globulus)
```

<https://github.com/famuvie/breedR/wiki/Overview>



# Blupf90 programs

## Practice

**blupf90**

BLUP with explicit equations

**remlf90**

Expectation Maximization REML

**aireml90**

Average Information REML

**gibbsf90**

Bayesian Analyses – linear traits

**thrgibbsf90**

Bayesian Analyses – categorical traits

**postgibbsf90**

Post-analyses of Gibbs samples

**blupf90+**

**gibbsf90+**

**renumf90**

Renumbering + data QC

**preGSf90**

Processing of SNP data (QC + matrices)

**QCf90**

QC of large SNP data

**postGS90**

Estimation of SNP effects and GWAS

**predf90**

Prediction of GEBV based on SNP effects

**seekparent90**

Parentage verification (SNP and pedigree)

**predictf90**

Adjusted and predicted phenotypes + residuals



✖

✖

✖

**blup90iod**

**cblup90iod**

**accf90**

**accf90GS**

✖ No need for the renumbering process

## Practice

- Renumf90 parameter file

**renumf90**

- `renumf90 --help`
- `renumf90 --show-template`

- FAQ blupf90

<https://nce.ads.uga.edu/wiki/doku.php?id=faq>

```
DATAFILE
data3.txt
TRAITS
4
FIELDS_PASSED TO OUTPUT

WEIGHT(S)

RESIDUAL_VARIANCE
0.60
EFFECT
3 cross alpha
EFFECT
1 cross alpha
RANDOM
animal
FILE
ped3.txt
FILE_POS
1 2 3 0 0
SNP_FILE
snp3.2k
PED_DEPTH
0
(CO)VARIANCES
0.40
OPTION map_file mrkmap.txt
OPTION use_yams
```

$$y = sex + animal + e$$

$$\hat{\sigma}_u^2 = 0.4$$

$$\hat{\sigma}_e^2 = 0.6$$



## Practice

- Renumf90 parameter file

**renumf90**

- FAQ blupf90

<https://nce.ads.uga.edu/wiki/doku.php?id=faq>

```
DATAFILE
renf90.dat
NUMBER_OF_TRAITS
1
NUMBER_OF_EFFECTS
2
OBSERVATION(S)
1
WEIGHT(S)

EFFECTS: POSITIONS_IN_DATAFILE NUMBER_OF_LEVELS TYPE_OF_EFFECT[EFFECT NESTED]
2      2 cross
3      12010 cross
RANDOM_RESIDUAL_VALUES
0.60000
RANDOM_GROUP
2
RANDOM_TYPE
add_an_upginb
FILE
renadd02.ped
(CO)VARIANCES
0.40000
OPTION SNP_file snp3.2k
OPTION map_file mrkmap.txt
OPTION use_yams
```

$$y = sex + animal + e$$
$$\hat{\sigma}_u^2 = 0.4$$
$$\hat{\sigma}_e^2 = 0.6$$



## Practice

### RANDOM\_GROUP

Number of the effect(s) from list of effects  
Correlated effects should be consecutive e.g. Maternal effects, Random Regression

### RANDOM\_TYPE

diagonal, add\_animal, add\_sire, add\_an\_upg,  
add\_an\_upginb, add\_an\_self, user\_file, user\_file\_i, or  
par\_domin

### FILE

Pedigree file, parental dominance, or user file

### (CO)VARIANCES

Square matrix with dimension equal to the  
 $\text{number\_of\_traits} * \text{number\_of\_correlated\_effects}$

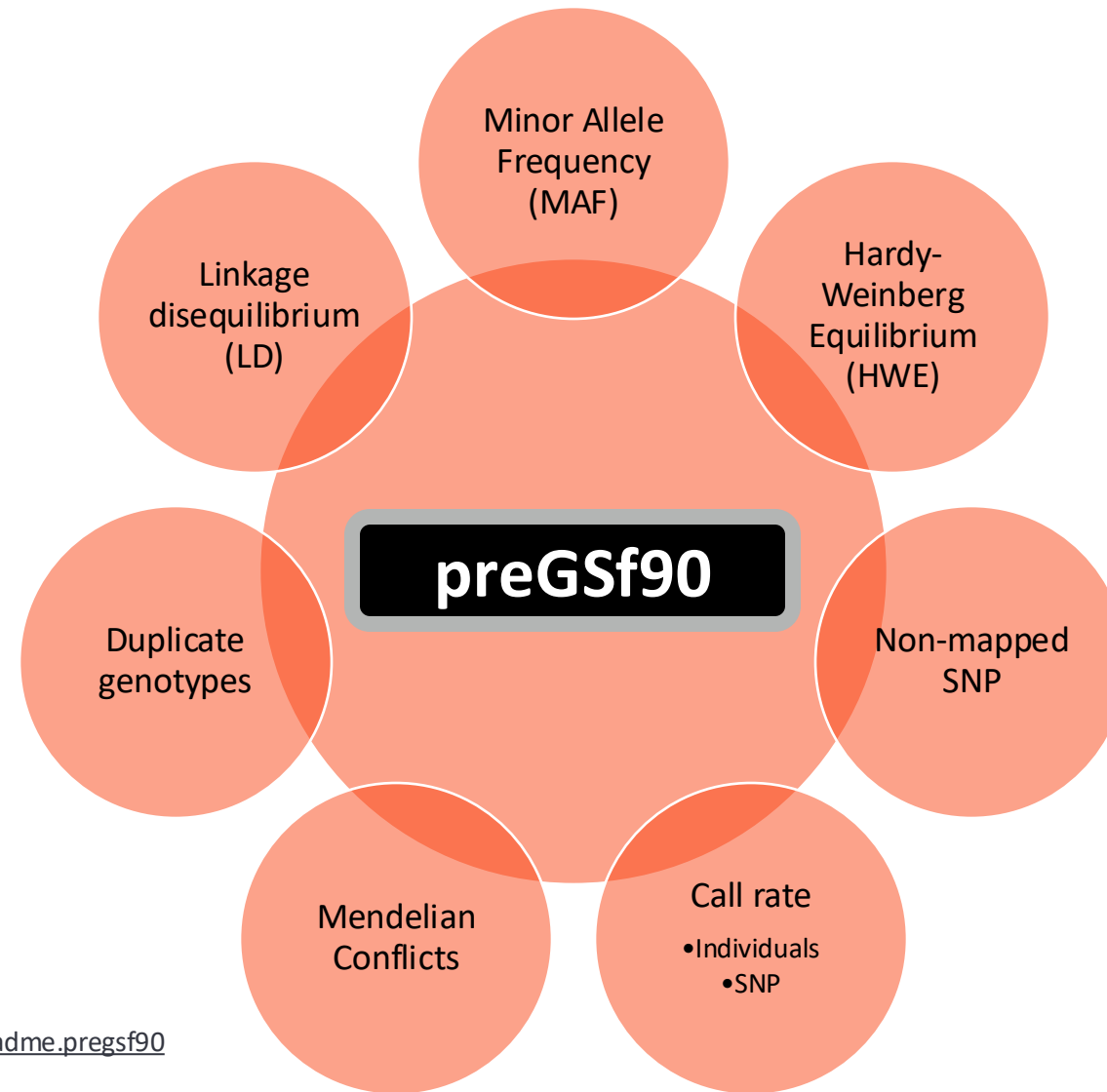
- *Add\_an\_self*
  - *To create a relationship matrix when there is selfing*
  - Pedigree file:
    - individual, parent 1, parent 2,  
number of selfing generations
- *user\_file*
  - An inverted matrix is read from the file
  - Matrix is stored only upper- or lower-triangular
  - Matrix file:
    - row, col, value
- *user\_file\_i*
  - As before but the matrix will be inverted by the program



## *preGSf90*

# Practice

- Quality control



<https://nce.ads.uga.edu/wiki/doku.php?id=readme.pregsf90>



## *preGSf90* Practice

- Same parameter file as for all BLUPF90 programs
- Needs an extra OPTION in renf90.par
- OPTION SNP\_file marker.geno
- Reads 2 extra files (besides data and pedigree):
  - marker.geno
  - marker.geno\_XrefID(created by renumf90)
- `_XrefID` has 2 columns: Renumbered ID Original ID

**preGSf90**



## Practice

- preGSf90 saves  $\mathbf{G}^{-1} - \mathbf{A}_{22}^{-1}$  by default (file: GimA22i)
  - To save 'raw' genomic matrix:
  - OPTION saveG [all]
    - If the optional all is present all intermediate G matrices will be saved!!!
- To save  $\mathbf{G}^{-1}$
- OPTION saveGInverse
  - Only the final G, after blending, scaling, etc. is inverted !!!
- To save  $\mathbf{A}_{22}$  and inverse
- OPTION saveA22 and OPTION saveA22Inverse

preGSf90





## Practice

- renf90.par
- OPTION method VCE
- OPTION EM-REML xx

**blupf90+**

**gibbsf90+**

```
DATAFILE
renf90.dat
NUMBER_OF_TRAITS
  1
NUMBER_OF_EFFECTS
  2
OBSERVATION(S)
  1
WEIGHT(S)

EFFECTS: POSITIONS_IN_DATAFILE NUMBER_OF_LEVELS TYPE_OF_EFFECT[EFFECT NESTED]
2      2 cross
3      12010 cross
RANDOM_RESIDUAL_VALUES
0.60000
RANDOM_GROUP
  2
RANDOM_TYPE
add_an_upginb
FILE
renadd02.ped
(CO)VARIANCES
0.40000
OPTION SNP_file snp3.2k_clean
OPTION map_file mrkmap.txt_clean
OPTION use_yams
```

$$y = sex + animal + e$$
$$\hat{\sigma}_u^2 = 0.4$$
$$\hat{\sigma}_e^2 = 0.6$$



## Practice

- EM-REML
  - OPTION SNP\_file snp3.2k\_clean
  - OPTION map\_file mrkmap.txt\_clean
  - OPTION no\_quality\_control
  - OPTION use\_yams
  - OPTION method VCE
  - OPTION EM-REML

At round: 23 Converge in fewer rounds than EM-REML rounds: 10000  
Stop EM-REML at 23 and no runs with AI-REML

\* END ITERATION: 07-17-2024 09h 54m 06s 649  
solutions stored in file: "solutions"

Final Estimates

Genetic variance(s) for effect 2

0.35532

Residual variance(s)

0.61222

\*\*\* Statistical Method: VCE

\* FINISHED (BLUPF90): 07-17-2024 09h 54m 06s 680

**blupf90+**



## Practice

- AI-REML
  - OPTION SNP\_file snp3.2k\_clean
  - OPTION map\_file mrkmap.txt\_clean
  - OPTION no\_quality\_control
  - OPTION use\_yams
  - OPTION method VCE

**blupf90+**

```
-2logL = 26720.6457620796 : AIC = 26724.6457620796
In round      4 convergence= 7.833323538291451E-014
delta convergence= 7.908716592526159E-008
new R
0.61221
new G
0.35534
* END ITERATION: 07-17-2024 10h 14m 55s 278
solutions stored in file: "solutions"
```

### Final Estimates

Genetic variance(s) for effect 2

0.35534

Residual variance(s)

0.61221

inverse of AI matrix (Sampling Variance)

0.73121E-03 -0.37380E-03

-0.37380E-03 0.32167E-03

Correlations from inverse of AI matrix

1.0000 -0.77076

-0.77076 1.0000

SE for G

0.27041E-01

SE for R

0.17935E-01

\*\*\* Statistical Method: VCE

\* FINISHED (BLUPF90): 07-17-2024 10h 14m 55s 315



## Practice

- AI-REML  
SE for genetic parameters

- `OPTION se_covar_function h2 G_2_2_1_1/(G_2_2_1_1+R_1_1)`

Notation is with reference to the effect number and the trait number (`G_eff1_eff2_trt1_trt2`) that indicate the element of the (co)variance matrix for random effect `eff1` and `eff2` and `trt1` and `trt2`, where `eff1` and `eff2` are effect numbers 1 and 2, and `trt1` and `trt2` are trait numbers 1 and 2. `R_trt1_trt1` indicates the element of the residual (co)variance matrix for traits 1 and 2.

**blupf90+**

- <https://nce.ads.uga.edu/wiki/doku.php?id=readme.pregsf90>



## Practice

- MME solver (default)
  - OPTION SNP\_file snp3.2k\_clean
  - OPTION map\_file mrkmap.txt\_clean
  - OPTION no\_quality\_control
  - OPTION use\_yams
  - OPTION store\_accuracy **eff** orig

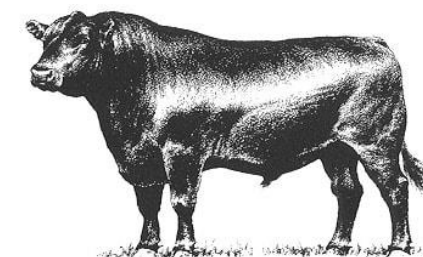
```
jorgehidalgo@endpoint-10-192-53-192 Data % head acc_bf90
trait/effect level original_id solution acc
1 2 1 UGA46217 0.05314548 0.5257
1 2 2 UGA46272 -0.16554279 0.5903
1 2 3 UGA43455 -1.22049127 0.5542
1 2 4 UGA51333 -0.22292902 0.5449
1 2 5 UGA42183 -0.15143591 0.7176
1 2 6 UGA51501 -0.09200698 0.5224
1 2 7 UGA43704 -0.12728916 0.5011
1 2 8 UGA44900 0.49888989 0.5319
1 2 9 UGA45303 -0.24224250 0.5009
```

**blupf90+**



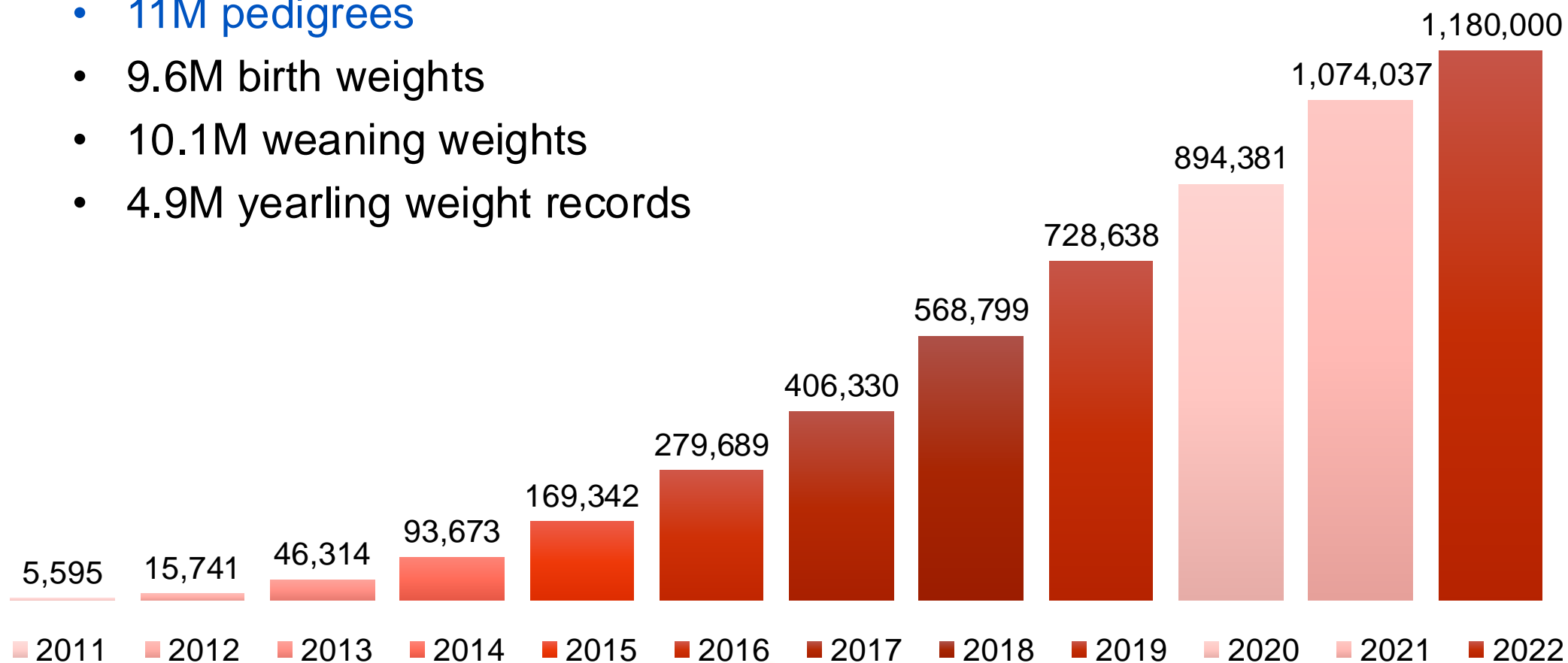
## An application example

### American Angus



- 11M pedigrees
- 9.6M birth weights
- 10.1M weaning weights
- 4.9M yearling weight records

# Genotyped Animals



## An application example – largest ssGBLUP evaluation



- US Holstein Type trait data
  - 18 trait-model
  - 13.6M animals in pedigree
  - 10.2M phenotypes
  - 2.3M genotyped animals
  - 447,492,870 equations to solve
- APY ssGBLUP with 15k core
  - 1 day to build  $\mathbf{G}_{APY}^{-1}$  and  $\mathbf{A}_{22}^{-1}$
  - ~ 2.5 days to converge
  - < 500 GB RAM with APY
- > 30 TB RAM to compute  $\mathbf{G}^{-1}$  without APY



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<https://doi.org/10.3168/jds.2020-18668>

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This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>).

### Bias in genomic predictions by mating practices for linear type traits in a large-scale genomic evaluation

S. Tsuruta,<sup>1\*</sup>  T. J. Lawlor,<sup>2</sup>  D. A. L. Lourenco,<sup>1</sup>  and I. Misztal<sup>1</sup> 

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<sup>2</sup>Holstein Association USA Inc., Brattleboro, VT 05301




# An application example

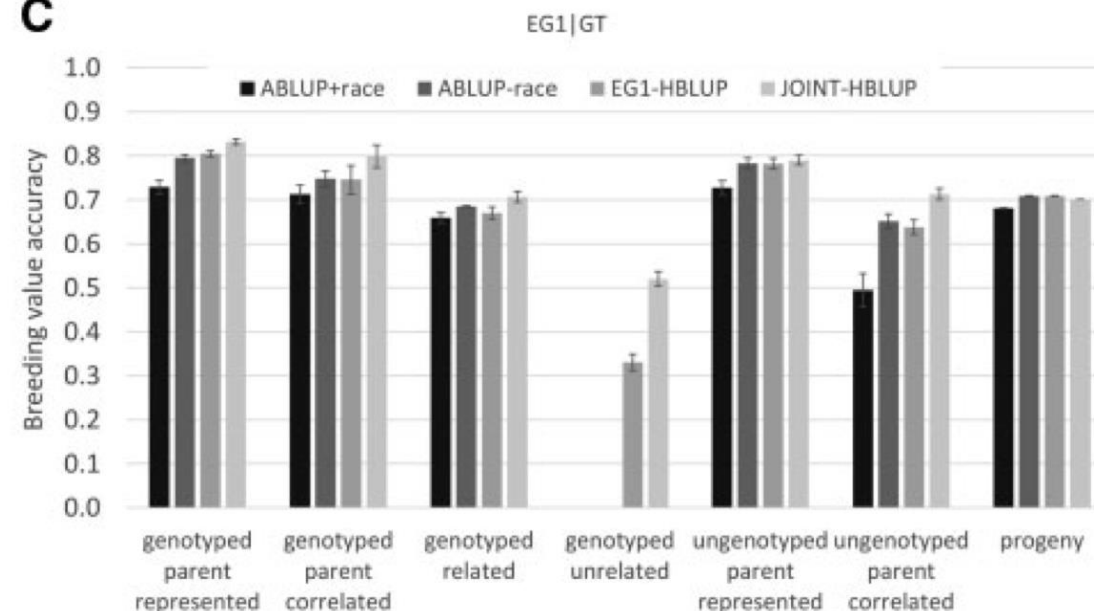


G3, 2021, 11(10), jkab253  
DOI: 10.1093/g3journal/jkab253  
Advance Access Publication Date: 16 July 2021  
Multiparental Populations

## Single-step genomic BLUP enables joint analysis of disconnected breeding programs: an example with *Eucalyptus globulus* Labill.

Andrew N. Callister <sup>1,\*</sup> Ben P. Bradshaw,<sup>2</sup> Stephen Elms,<sup>3</sup> Ross A. W. Gillies,<sup>3</sup> Joanna M. Sasse,<sup>4</sup> and Jeremy T. Brawnner<sup>5</sup>

C





# *Blupf90 programs*

## Practice

### Bases for Genomic Prediction

Andres Legarra

Daniela A.L. Lourenco

Zulma G. Vitezica

2024-02-21



<https://genoweb.toulouse.inra.fr/~alegarra/GSIP.pdf>



# Thanks!

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[http://nce.ads.uga.edu/html/projects/programs/docs/blupf90\\_all8.pdf](http://nce.ads.uga.edu/html/projects/programs/docs/blupf90_all8.pdf)

